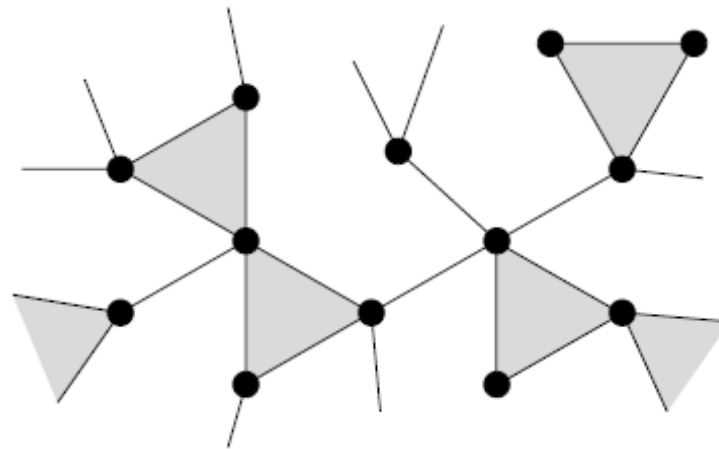


# Economic systems in and out of equilibrium: network models of trade, systemic risk, and early-warning signals



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INSTITUUT  
LORENTZ

# Part 1:

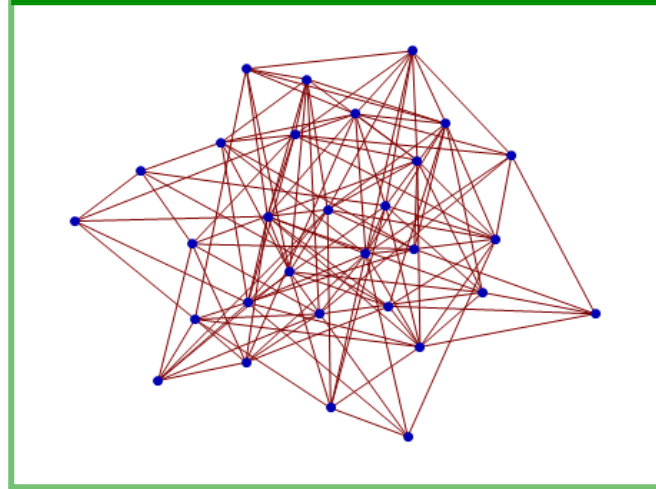
checking for (quasi-)equilibrium  
via **Maximum-Entropy Ensembles**

# Complex (economic) systems over long times => in or out of equilibrium?

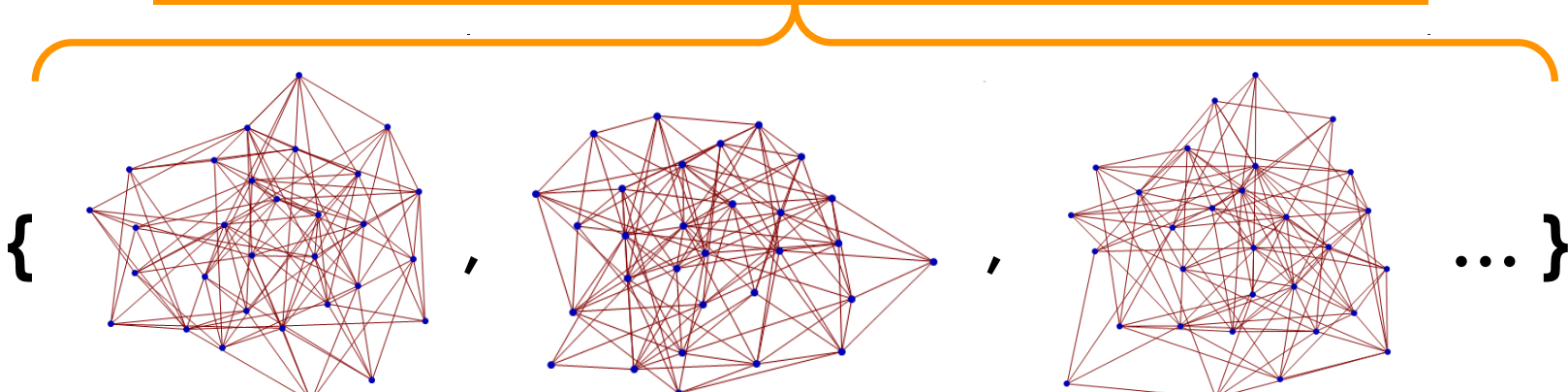
- Large complex systems: direct **microscopic** description **impossible** and maybe **noisy** (e.g. like particles in a room);
- Identify robust **macroscopic** properties (e.g. total energy), assume all the rest is random (**pause economic theory**);
- **Construct equilibrium model**: treat the macroscopic properties as **constraints**, maximize the **entropy** and make **inference** on the microscopic state;
- Redo for **multiple snapshots**: If **higher-order** (w.r.t. constraints) properties are correctly replicated, your system is **quasi-equilibrium**
- **Release theory**: check whether the constraint (e.g. energy) is controlled by **economic factors** (e.g. temperature): if so, you have a **functionally explicit microscopic model** with an **explanatory variable!**

# Constructing (quasi-)equilibrium ensembles

Real system



Null model (equilibrium ensemble)



T. Squartini and D. Garlaschelli, *New. J. Phys.* **13**, 083001 (2011)

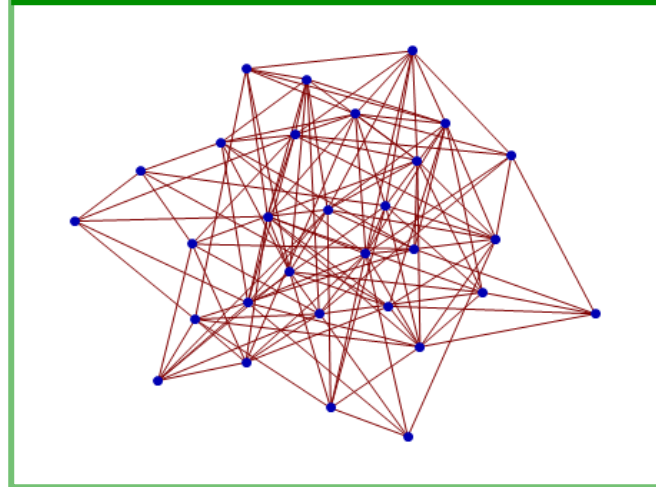
# Constructing (quasi-)equilibrium ensembles

Maximize the  
**entropy**

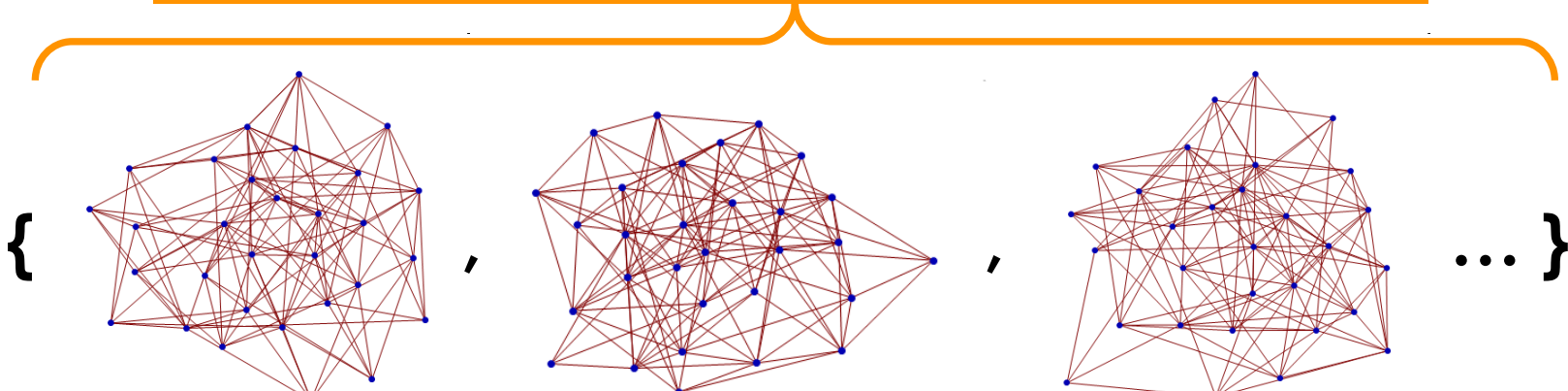
$$S \equiv - \sum_{\mathbf{G}} P(\mathbf{G}) \ln P(\mathbf{G})$$

subject to some  
good **constraint C**

**Real system**



**Null model (equilibrium ensemble)**



T. Squartini and D. Garlaschelli, *New. J. Phys.* **13**, 083001 (2011)

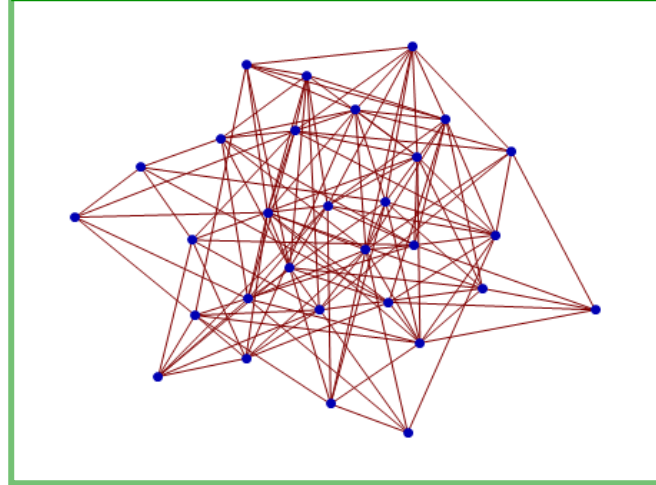
# Constructing (quasi-)equilibrium ensembles

Maximize the  
**entropy**

$$S \equiv - \sum_{\mathbf{G}} P(\mathbf{G}) \ln P(\mathbf{G})$$

subject to some  
good **constraint C**

**Real system**

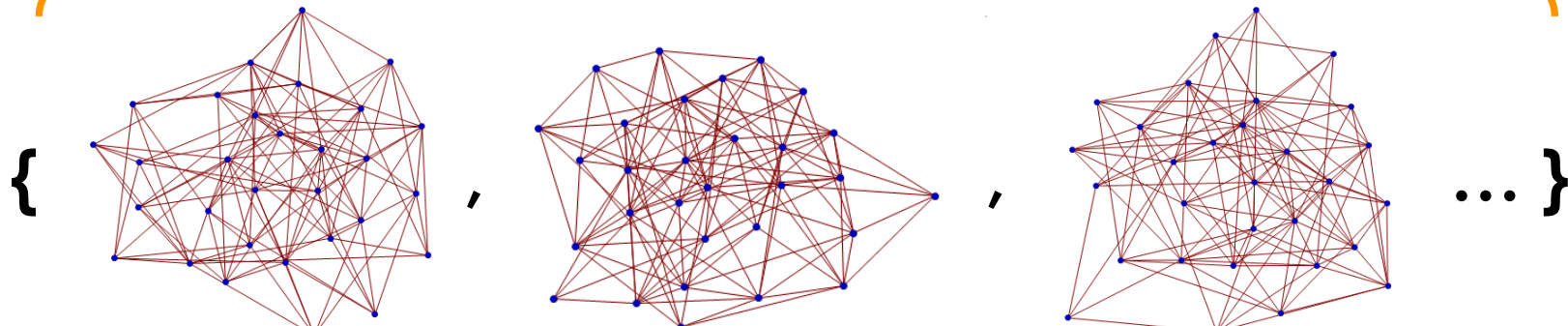


**Check** for equilibrium  
via deviations

$$z_X \equiv \frac{X - \langle X \rangle}{\sigma[X]}$$

of **higher-order**  
properties **X**

**Null model (equilibrium ensemble)**

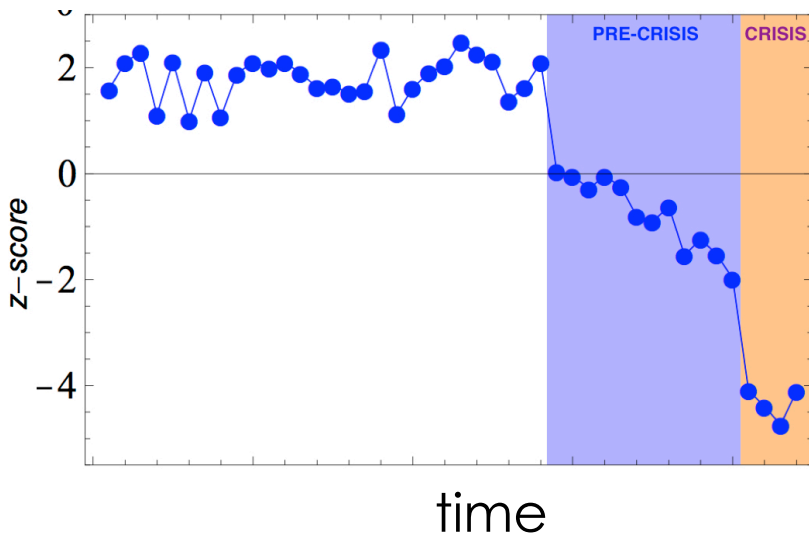


T. Squartini and D. Garlaschelli, *New. J. Phys.* **13**, 083001 (2011)

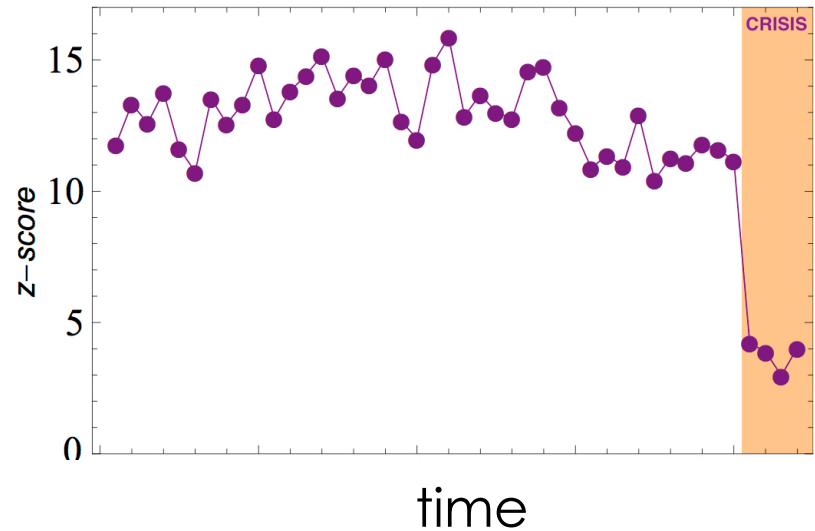
# Example:

same system, two choices of constraints

## Choice 1



## Choice 2

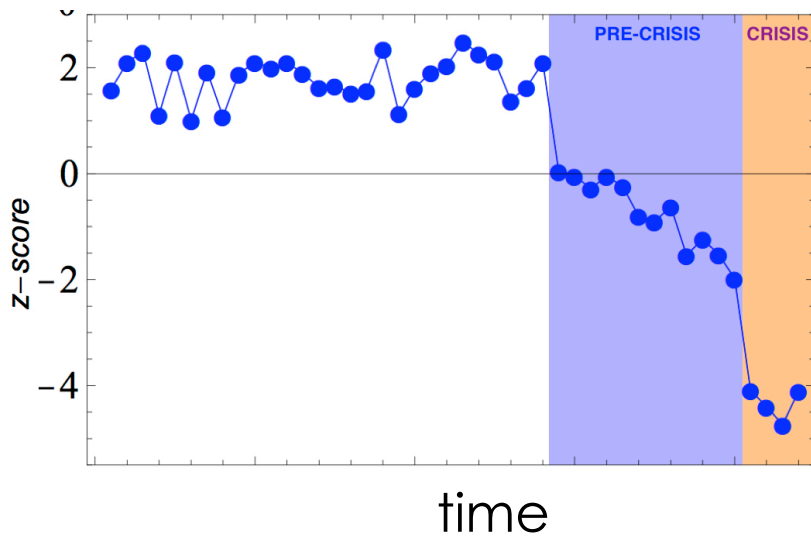


Transition **from quasiequilibrium of model 1** (left panel, white) **to quasi-equilibrium of model 2** (right panel, orange) via a **non-equilibrium** regime (blue): **early-warning signal!**

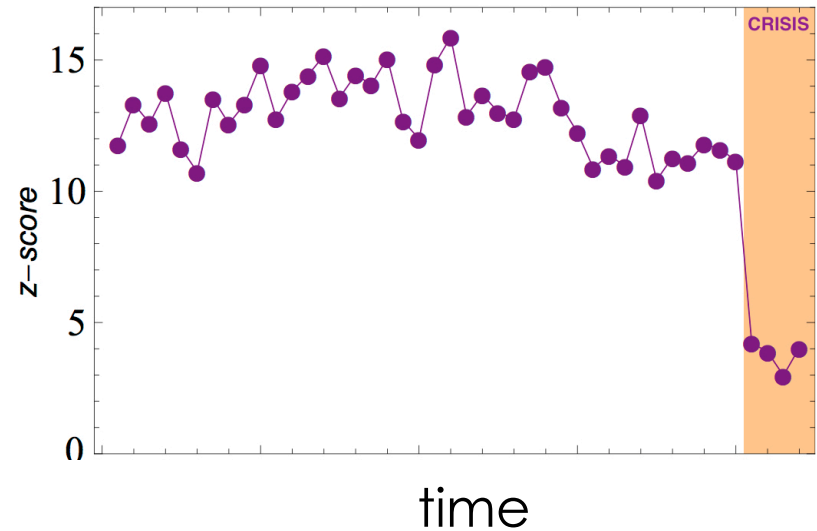
# Example:

same system, two choices of constraints

## Choice 1



## Choice 2



Transition **from quasiequilibrium of model 1** (left panel, white) **to quasi-equilibrium of model 2** (right panel, orange) via a **non-equilibrium** regime (blue): **early-warning signal!**

**Real example: Dutch interbank network 1998-2008!**



## COMPLEX SYSTEMS

# Complexity theory and financial regulation

Economic policy needs interdisciplinary network analysis and behavioral modeling

By Stefano Battiston,<sup>1\*</sup> J. Dooyne Farmer,<sup>2,3</sup> Andreas Flache,<sup>4</sup> Diego Garlaschelli,<sup>5</sup> Andrew G. Haldane,<sup>6</sup> Hans Heesterbeek,<sup>7</sup> Cars Hommes,<sup>8,9\*</sup> Carlo Jaeger,<sup>10,11,12</sup> Robert May,<sup>13</sup> Marten Scheffer<sup>14</sup>

Traditional economic theory could not explain, much less predict, the near collapse of the financial system and its long-lasting effects on the global economy. Since the 2008 crisis, there has been increasing interest in using ideas from complexity theory to make sense of economic and financial markets. Concepts, such as tipping points, networks, contagion, feedback, and resilience have entered the financial and regulatory lexicon, but

**POLICY** actual use of complexity models and results remains at an early stage. Recent insights and techniques offer potential for better monitoring and management of highly interconnected economic and financial systems and, thus, may help anticipate and manage future crises.

**TIPPING POINTS, WARNING SIGNALS.** Financial markets have historically exhibited sudden and largely unforeseen collapses, at a systemic scale. Such “phase transitions” may in some cases have been triggered by unpredictable stochastic events. More often, however, there have been endogenous underlying processes at work. Analyses of complex systems ranging from the climate to ecosystems reveal that, before a major transition, there is often a gradual and unnoticed loss of resilience. This makes the system brittle: A small disruption can trigger a domino effect that propagates through the system and propels it into a crisis state.

Recent research has revealed generic empirical quantitative indicators of resilience that may be used across complex systems to detect tipping points. Markers include rising correlation between nodes in a network and rising temporal correlation, variance, and skewness of fluctuation patterns. These indicators were first predicted mathematically and subsequently demonstrated experimentally in real complex systems, including living systems (1). A recent study of the Dutch interbank network (2) showed that standard analysis using a homogeneous network model could only lead to late detection of the 2008 crisis, although a more realistic and heterogeneous network model could identify an early warning signal 3 years before the crisis (see the chart).

Ecologists have developed tools to quantify the stability, robustness, and resilience of food webs and have shown how these depend on the topology of the network and the strengths of interactions (3). Epidemiologists have tools to gauge the potential for events to propagate in systems of interacting entities, to identify superspreaders and core groups relevant to infection persistence, and to design strategies to prevent or limit the spread of contagion (4).

Extrapolating results from the natural sciences to economics and finance presents challenges. For instance, publication of an early warning signal will change behavior and affect future dynamics [the Lucas critique (5)]. But this does not affect the case where indicators are known only to regulators or when the goal is to build better network barriers to slow contagion.

**TOO CENTRAL TO FAIL.** Network effects matter to financial-economic stability because shock amplification may occur via strong cascading effects. For example, the Bank of International Settlements recently developed a framework drawing on data on the interconnectedness between banks to gauge the systemic risk posed to the financial network by Global Systemically Important Banks. Recent research on contagion in financial networks has shown that network topology and positions of banks matter; the global financial network may collapse even when individual banks appear safe (6). Capturing these effects is essential for quantifying stress on individual banks and for looking at systemic risk for the network as

a whole. Despite on-going efforts, these effects are unlikely to be routinely considered anytime soon.

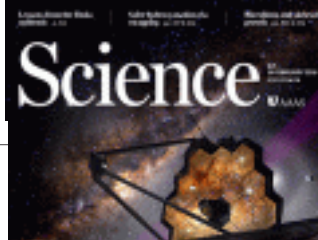
Information asymmetry within a network—e.g. where a bank does not know about troubled assets of other banks—can be problematic. The banking network typically displays a core-periphery structure,

**“...policies and financial regulation [that] weaken positive feedback... stabilize experimental macroeconomic systems...”**

with a core consisting of a relatively small number of large, densely interconnected banks that are not very diverse in terms of business and risk models. This implies that core banks' defaults tend to be highly correlated. That, in turn, can generate a collective moral hazard problem (i.e., players take on more risk, because others will bear the costs in case of default), as banks recognize that they are likely to be supported by the authorities in situations of distress, the likelihood amplifies their incentives to herd in the first place.

Estimating systemic risk relies on granular data on the financial network. Unfortunately, business interactions between banks are often hidden because of confidentiality issues. Tools being developed to reconstruct networks from partial information and to estimate systemic risk (7) suggest that publicly available bank information does not allow reliable estimation of systemic risk. The estimate would improve greatly if banks publicly reported the number of connections with other banks, even without disclosing their identity.

In addition to data, understanding the effects of interconnections also relies on integrative quantitative metrics and concepts that reveal important network aspects, such as systemic repercussions of the failure of individual nodes. For example, DebtRank, which measures the systemic importance of individual institutions in a financial network (8), shows that the issue of too-central-to-fail may be even more important than too-big-to-fail.



**AGENTS AND BEHAVIOR.** Agent-based models (ABMs) are computer models in which the behavior of agents and their interactions are explicitly represented as decision rules mapping agents' observations onto actions. Although ABMs are less well established in analyzing financial-economic systems than in, e.g., traffic control, epidemiology, or battlefield conflict analyses, they have produced promising results. Axtell (9) developed a simple ABM that explains more than three dozen empirical properties of firm formation without recourse to external shocks. ABMs provide a good explanation for why the volatility of prices is clustered and time-varying (10) and have been used

Laboratory experiments with human subjects can provide empirical validation of individual decision rules of agents, their interactions, and emergent macro behavior. Recent experiments studying behavior of a group of individuals in the laboratory show that economic systems may deviate significantly from rational efficient equilibrium at both individual and aggregate levels (14). This generic feature of positive feedback systems leads to persistent deviations of prices from equilibrium and emergence of speculation-driven bubbles and crashes, strongly amplified by coordination on trend-following and herding behavior (15). There is strong empirical evidence of

monetary and fiscal policies and financial regulation designed to weaken positive feedback are successful in stabilizing experimental macroeconomic systems when properly calibrated (16). Complexity theory provides mathematical understanding of these effects.

**POLICY DASHBOARD.** It is an opportune time for academic economists, complexity scientists, social scientists, ecologists, epidemiologists, and researchers at financial institutions to join forces to develop tools from complexity theory, as a complement to existing economic modeling approaches (17). One ambitious option would be an online, financial-economic dashboard that integrates data, methods, and indicators. This might monitor and stress-test the global socioeconomic and financial system in something close to real time, in a way similar to what is done with other complex systems, such as weather systems or social networks. The funding required for essential policy-relevant and fundamental interdisciplinary progress in these areas would be trivial compared with the costs of systemic financial failures or the collapse of the global financial-economic system. ■

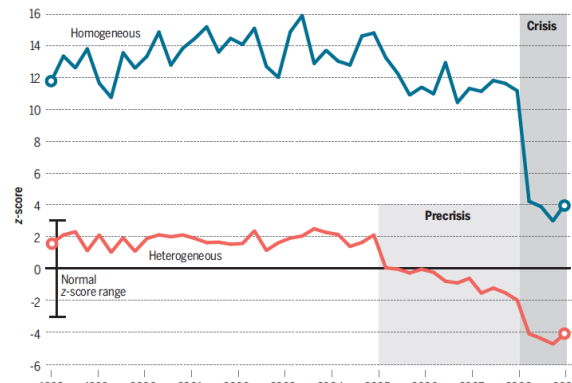
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10.1126/science.aad0299



**Early-warning signals of the 2008 crisis in the Dutch interbank network.** The figure portrays a temporal analysis of two loops, pairs of banks that are at the same time debtor and creditor to each other. Although the raw number of two loops is not very informative about possible ongoing structural changes, its comparison with a random network model benchmark is. A z-score represents the number of standard deviations by which the number of two loops in the real network deviates from its expected value in the model. Small magnitude z-scores (<3) indicate approximate consistency with the model, whereas larger magnitudes indicate statistically significant deviations. Two different random network models were used: a homogeneous network with the same total number of links as in the real network (top) and a heterogeneous network where every bank has the same number of connections as in the real network (bottom). The homogeneous model, often used in standard analyses, highlights only a late and abrupt structural change (2008). The more realistic heterogeneous model also identifies a gradual, early-warning “precrisis” phase (2005–2007). [Modified from (2)]

to test systemic risk implications of reforms developed by the Basel Committee on Banking Supervision, which show how dynamically changing risk limits can lead to booms and busts in prices (11, 12). ABMs of market dynamics can be linked with ABM work on opinion dynamics in the social sciences (13) to understand how propagation of opinions through social networks affects emergent macro behavior, which is crucial to managing the stability and resilience of socioeconomic systems.

These behaviors in financial markets in practice, and these controlled laboratory experiments provide more detailed understanding of mechanisms, causality, and conditions for emergence of macro phenomena.

A simple behavioral model, with agents gradually switching to better performing heuristics, explains individual, as well as emergent, macro behavior in these laboratory economies. The experiments also provide a general mechanism for managing social contagion in such systems. For example,

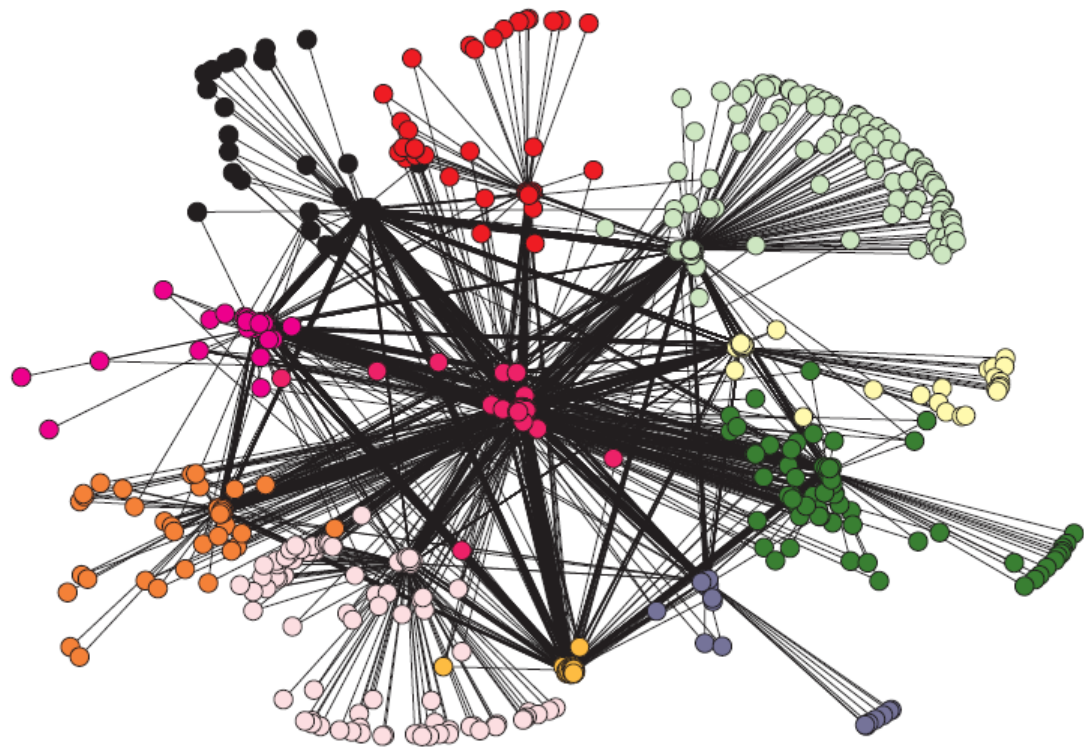
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## Part 2:

if systems are at (quasi-)equilibrium,  
their structure can be  
**reconstructed from  
partial information**

(i.e. from the “right” constraints)

# The challenge: reconstructing (interbank) networks from partial information



Crucial for estimating  
systemic risk:  
collapse of entire network

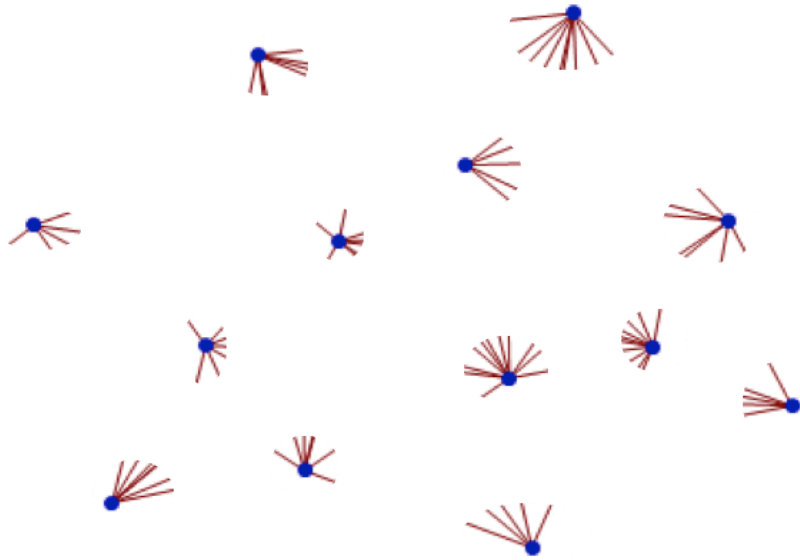
## Public:

each bank's total exposure towards the **aggregate** of all other banks

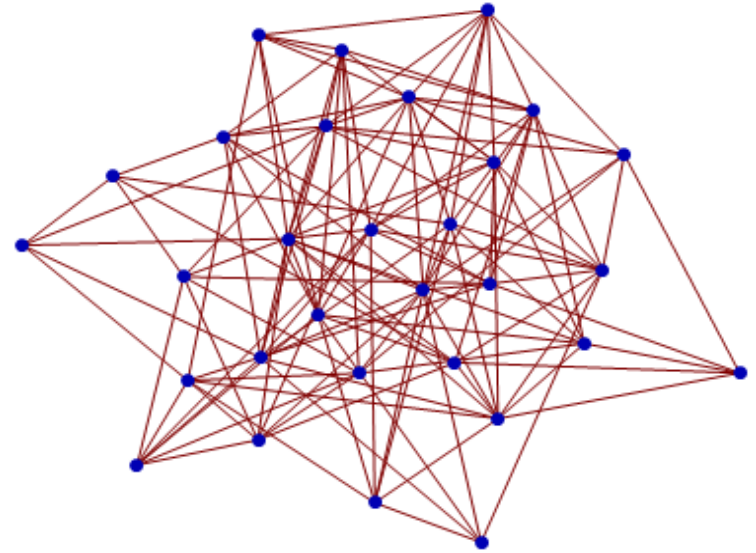
## Hidden:

each bank's individual exposure towards each **single** bank

**Local properties,  $O(N)$**   
(known/public)



**Original network,  $O(N^2)$**   
(unknown/hidden)



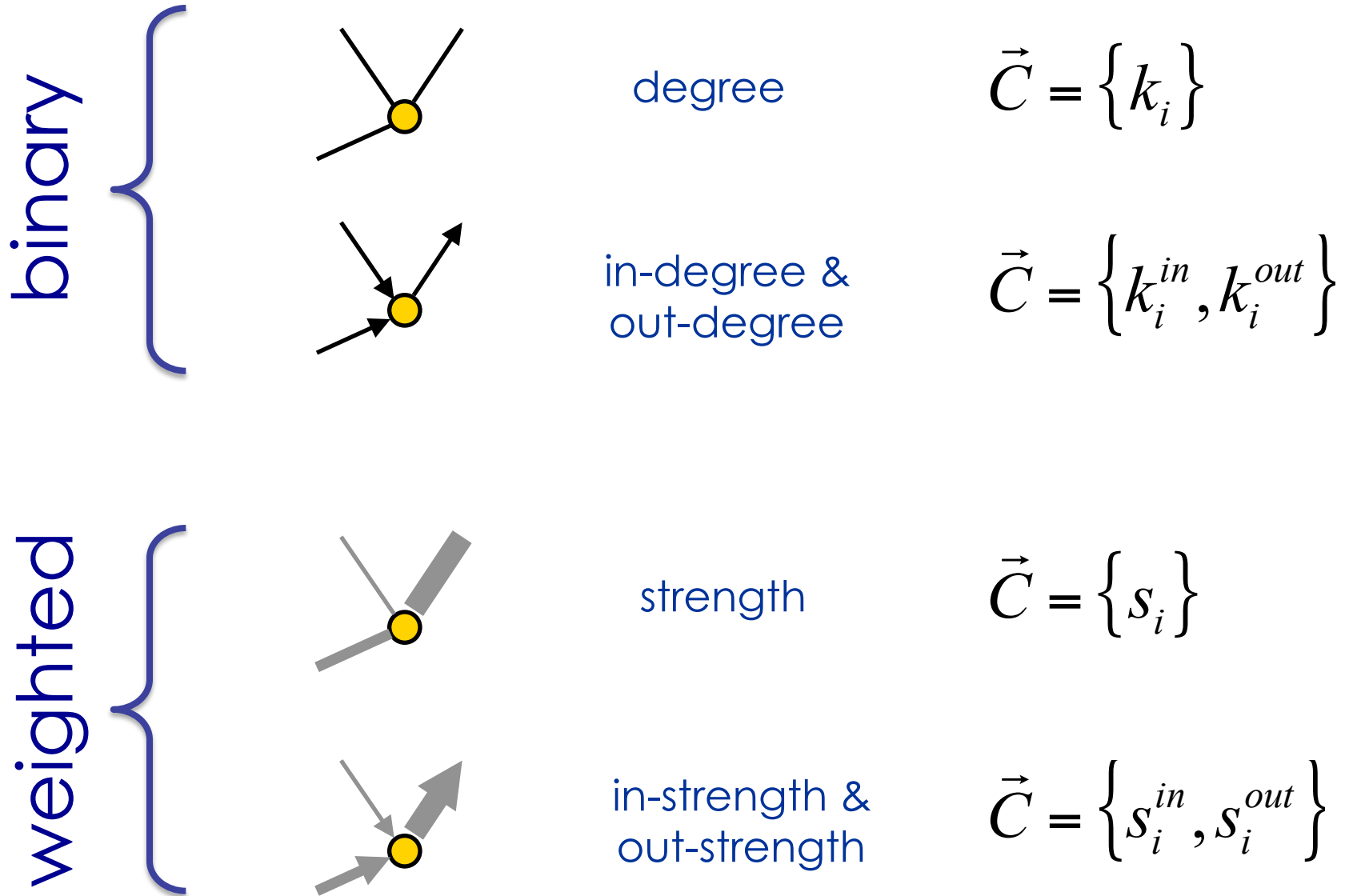
**Our goal:**

Can we **statistically reconstruct**  
the original structure in such a way that:

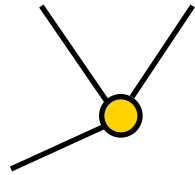
- 1) *Privacy is protected*
- 2) *Higher-order effects are correctly predicted*



# Reconstruction from **local** information (constraints)

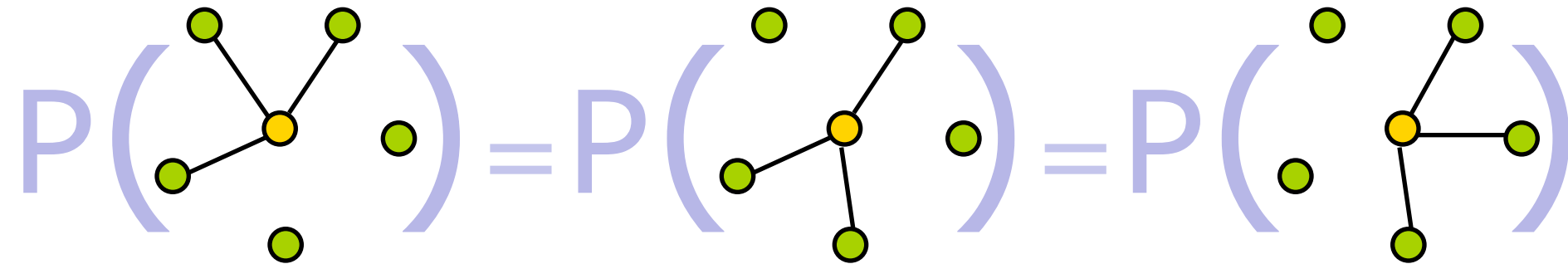


# Binary constraints: fixed **degree** sequence



$$\vec{C} = \{k_i\}$$

Equiprobable configurations:

$$P(\text{graph 1}) = P(\text{graph 2}) = P(\text{graph 3})$$


(must hold for all vertices simultaneously)

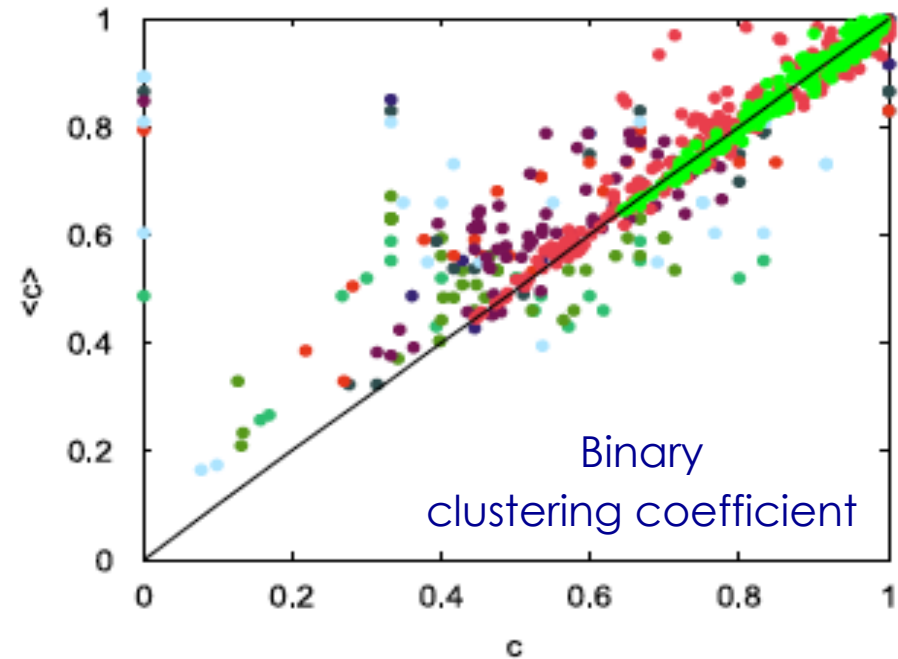
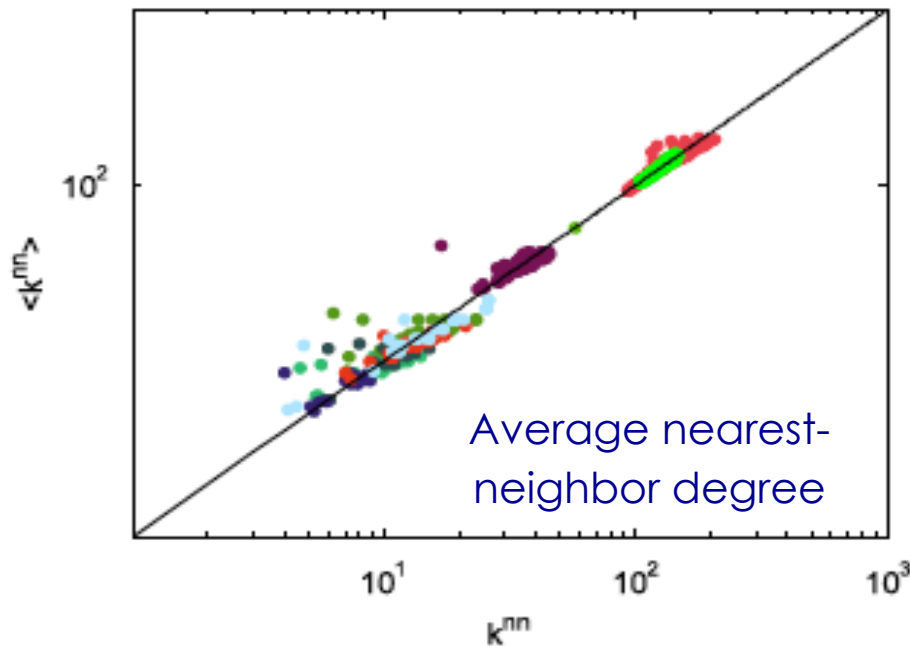
Note: the resulting distribution is FERMI-DIRAC

# Result:

good **binary** reconstruction  
of higher-order properties  
from degrees only

## Network

- Office social network [27]
- Research group social network [27]
- Fraternity social network [27]
- Maspalomas Lagoon food web [28]
- Chesapeake Bay food web [28]
- Crystal River (control) food web [28]
- Crystal River food web [28]
- Michigan Lake food web [28]
- Mondego Estuary food web [28]
- Everglades Marshes food web [28]
- Italian interbank network (1999) [26]
- World Trade Web (2000) [20]



# Weighted constraints: fixed **strength** sequence



Equiprobable configurations:

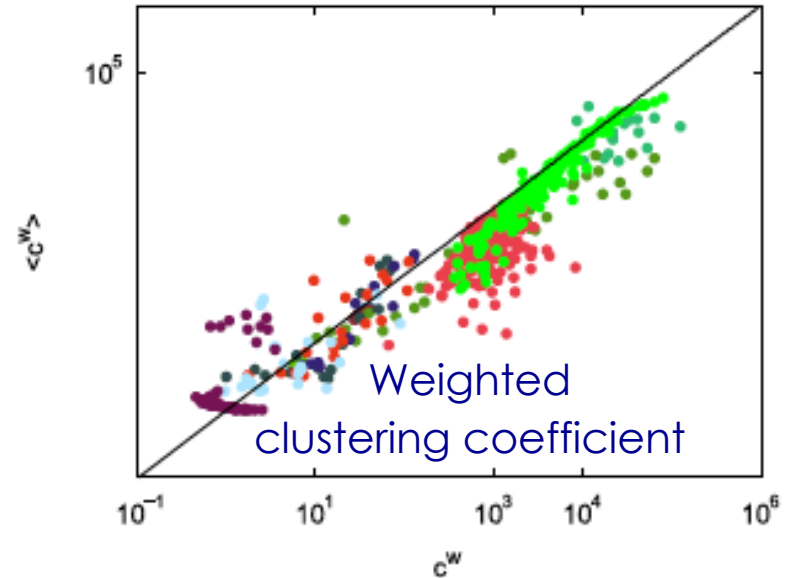
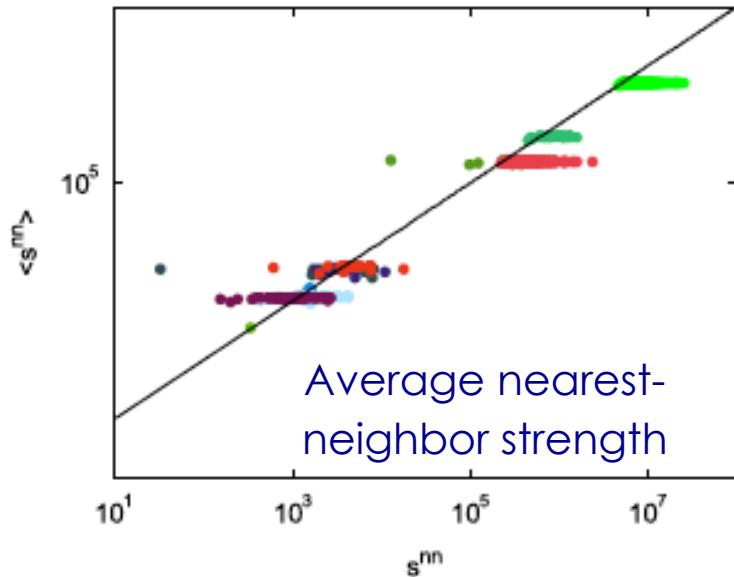
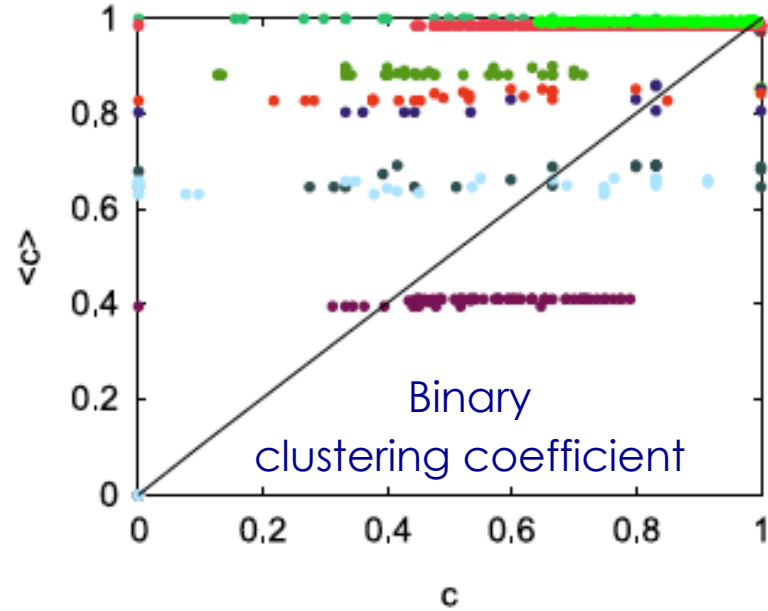
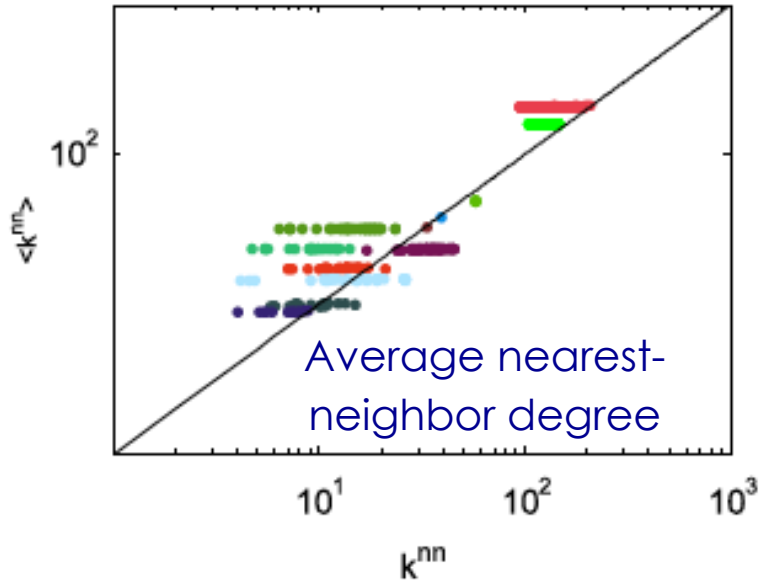
$$\begin{aligned} P\left(\begin{array}{c} 1 \\ \diagup \\ \text{---} \\ \diagdown \\ 2 \end{array} \begin{array}{c} \text{---} \\ \diagup \\ 3 \end{array}\right) &= P\left(\begin{array}{c} 1 \\ \text{---} \\ \diagup \\ 4 \end{array} \begin{array}{c} \text{---} \\ \diagdown \\ 1 \end{array}\right) = P\left(\begin{array}{c} \text{---} \\ \diagup \\ 3 \end{array} \begin{array}{c} \text{---} \\ \diagdown \\ 3 \end{array}\right) \\ &= P\left(\begin{array}{c} 2 \\ \text{---} \\ \diagup \\ 2 \end{array} \begin{array}{c} \text{---} \\ \diagdown \\ 2 \end{array}\right) = P\left(\begin{array}{c} \text{---} \\ \diagup \\ 6 \end{array}\right) \end{aligned}$$

(must hold for all vertices simultaneously)

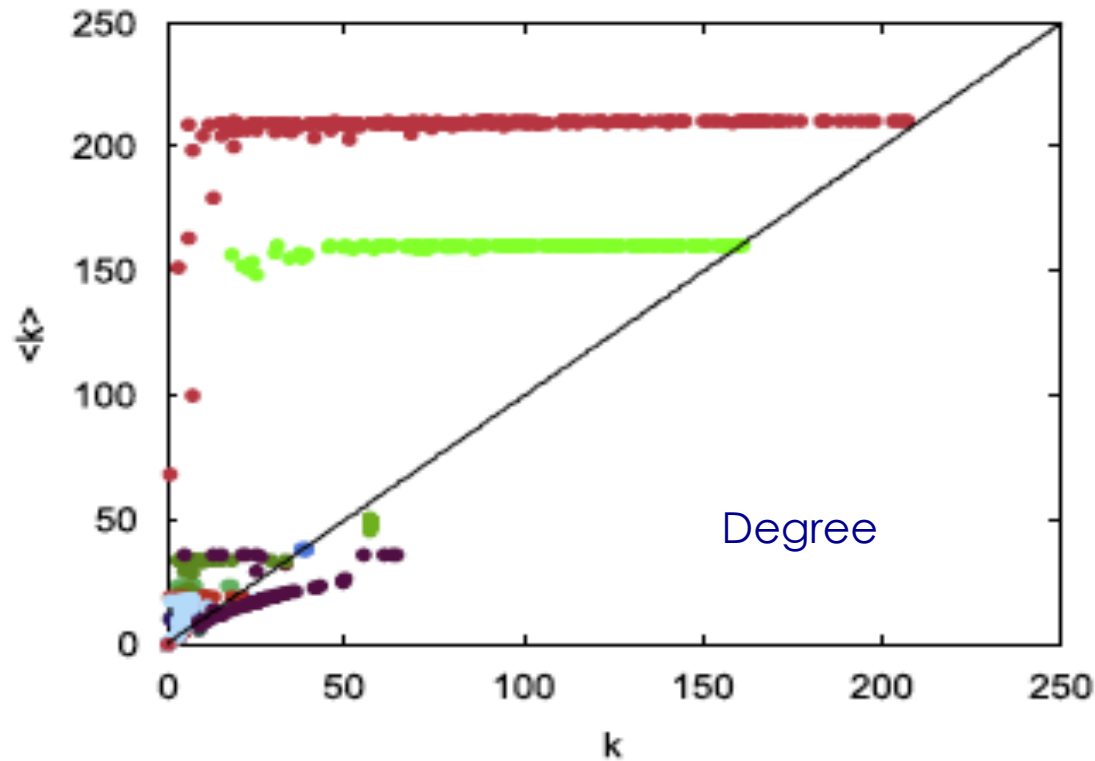
Note: the resulting distribution is BOSE-EINSTEIN



# Bad standard reconstruction (from strengths only)

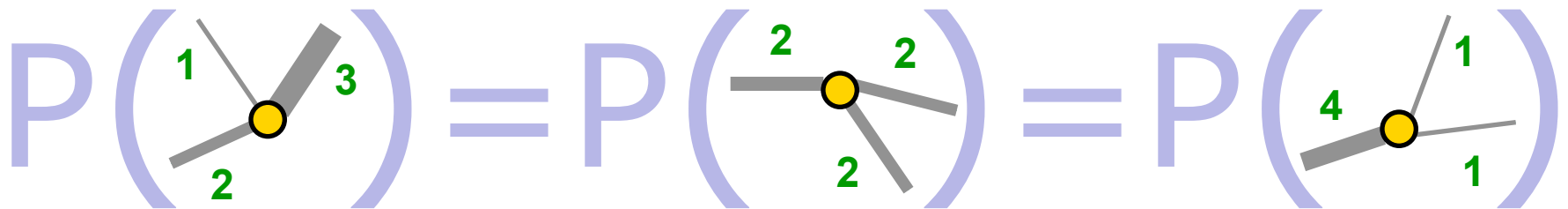
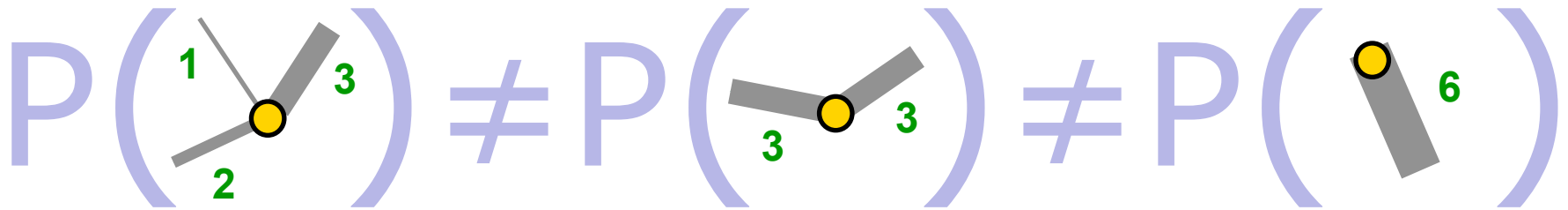
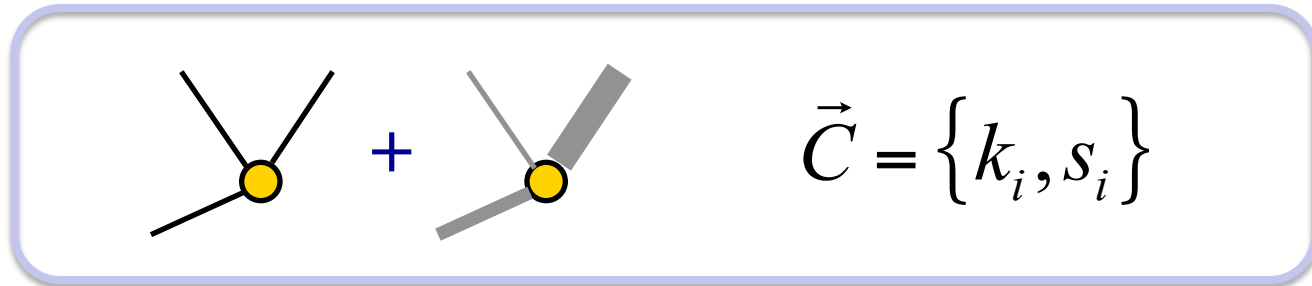


Reason: poor **binary** reconstruction from strengths only



The naive expectation that aggregate **weighted** properties are more informative than **binary** ones is **incorrect!**

# Doubling the constraints: degrees + strengths

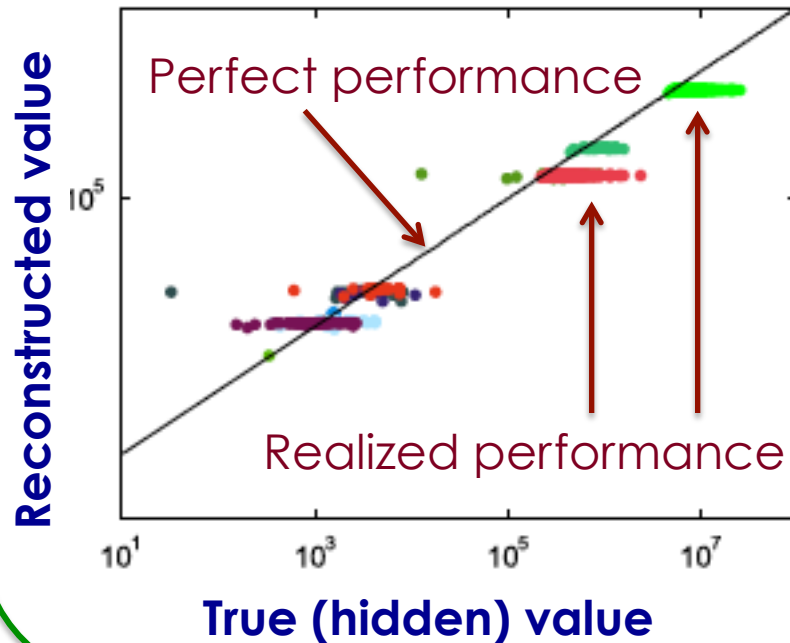
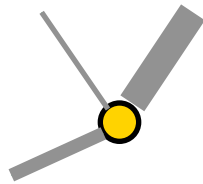


Note: the resulting distribution is BOSE-FERMI (mixed!)

# Example: reconstructing the average exposure of neighboring banks

## Traditional approach

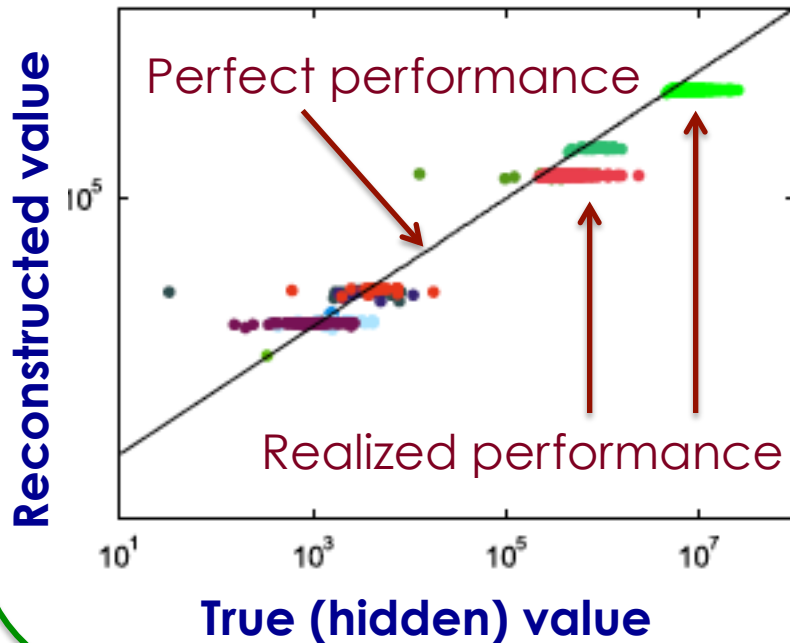
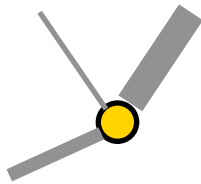
(from “strengths” only)



# Example: reconstructing the average exposure of neighboring banks

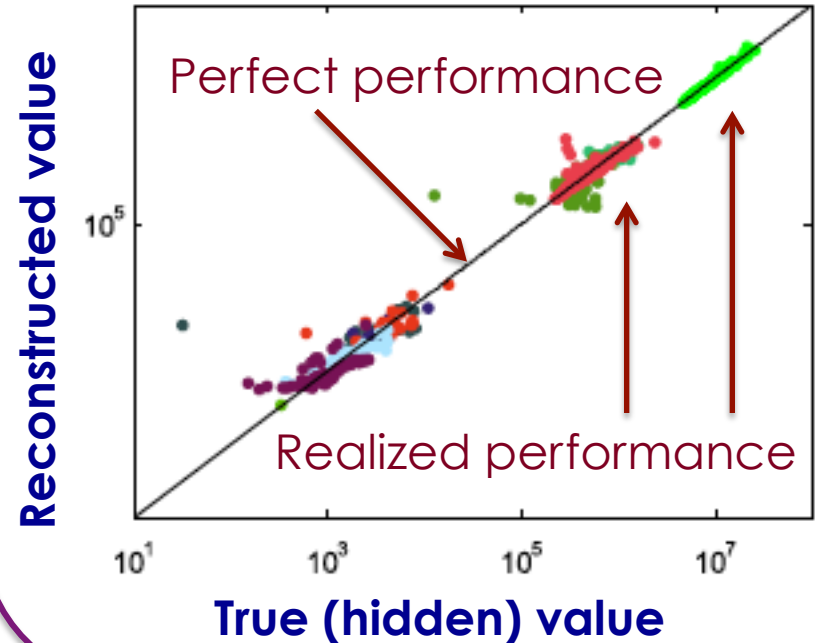
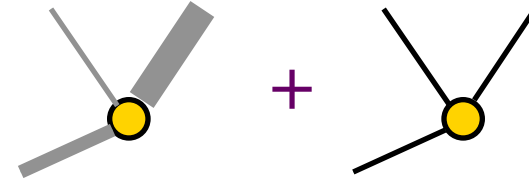
## Traditional approach

(from "strengths" only)

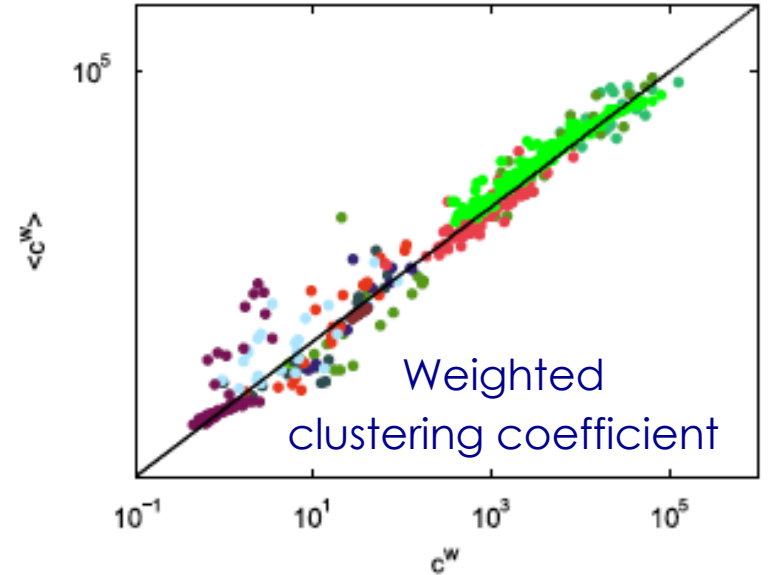
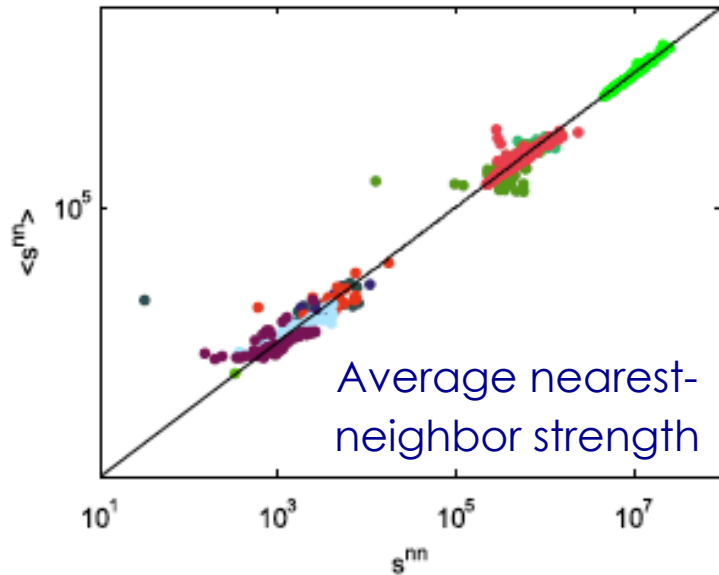
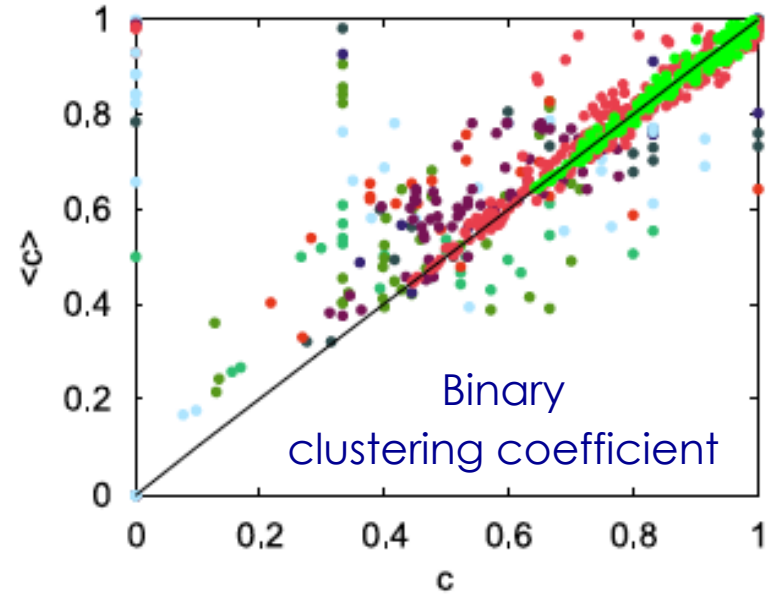
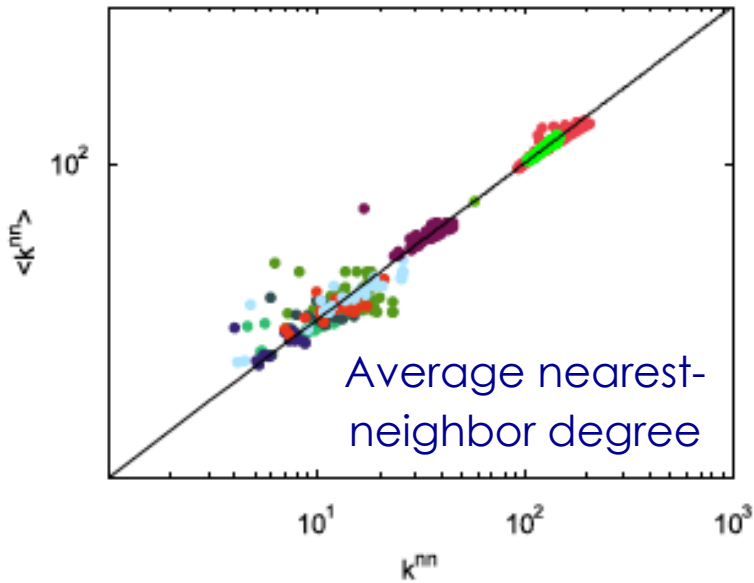


## Enhanced method

(from strengths + degrees)



# Enhanced reconstruction (from strengths and degrees)



# Reconstructing systemic risk estimators

## Percolation

(relative size of giant component vs occupation probability  $p$ )

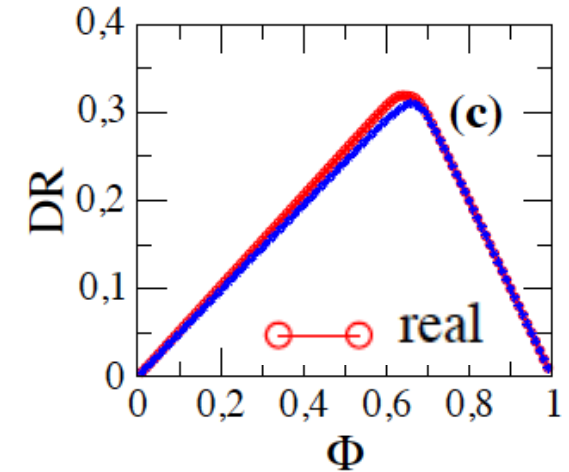
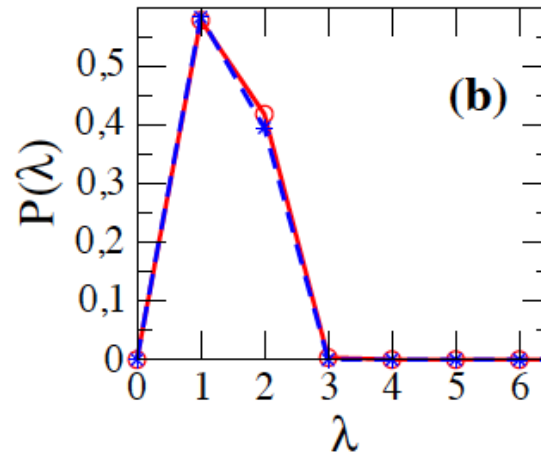
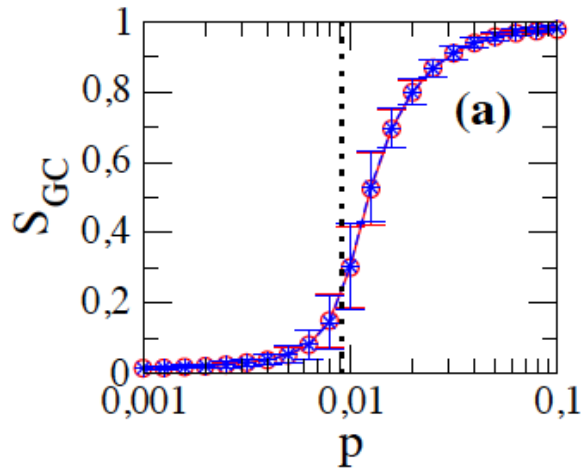
## Path length

(distribution of shortest distances  $\lambda$  among pairs of nodes)

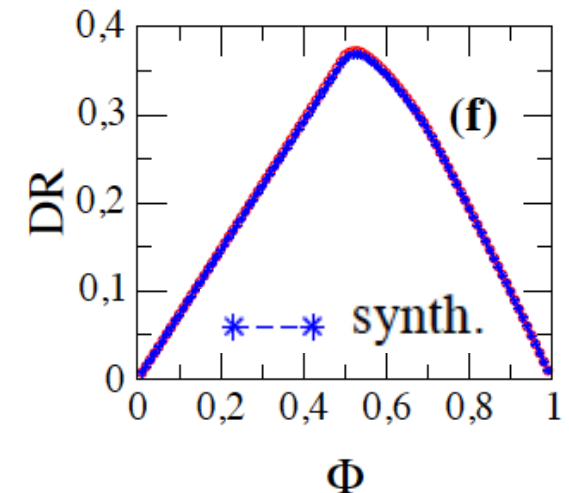
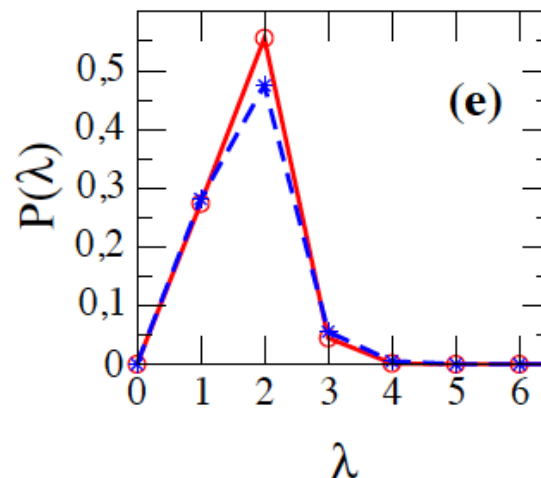
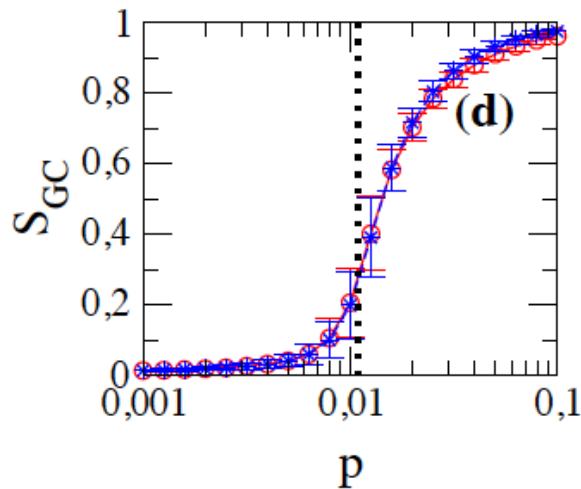
## Group DebtRank

(total devaluation induced by an initial devaluation  $\Phi$ )  
[Battiston et al. 2012]

WTW



E-mid



$w_{i \rightarrow j}$  : true (unknown)

$\tilde{w}_{i \rightarrow j}$  : reconstructed from margins  $\left\{ \begin{array}{l} s_i^{in} = \sum_{j \in V} w_{j \rightarrow i} \\ s_i^{out} = \sum_{j \in V} w_{i \rightarrow j} \end{array} \right.$

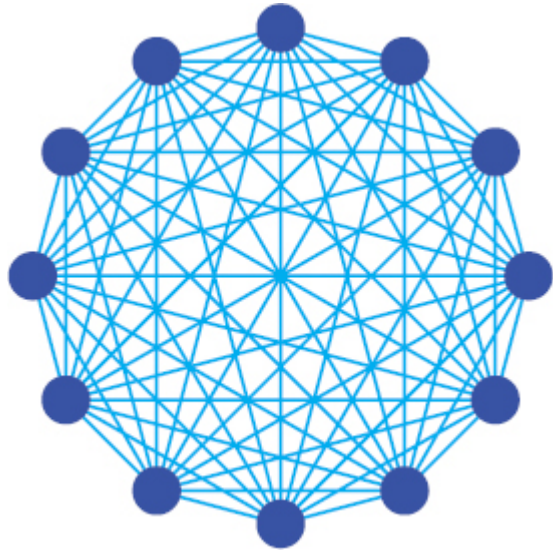


$W_{i \rightarrow j}$  : true (unknown)

$\tilde{W}_{i \rightarrow j}$  : reconstructed from margins  $\left\{ \begin{array}{l} s_i^{in} = \sum_{j \in V} w_{j \rightarrow i} \\ s_i^{out} = \sum_{j \in V} w_{i \rightarrow j} \end{array} \right.$

## Traditional approach

(from “strengths” only)



$$\tilde{W}_{i \rightarrow j} = \frac{s_i^{out} s_j^{in}}{W}$$

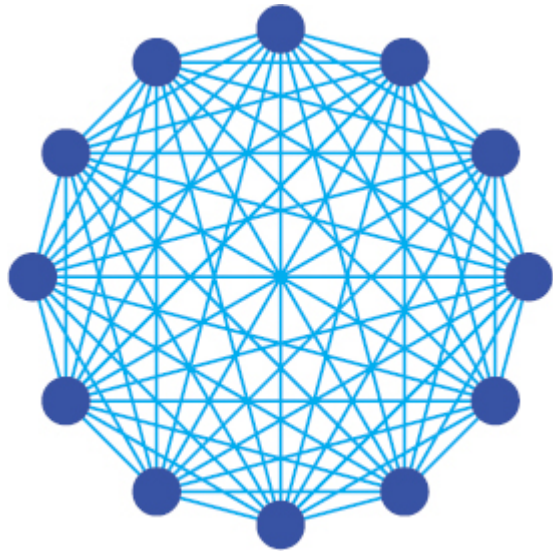
margins: **OK**, topology: **BAD**

$W_{i \rightarrow j}$  : true (unknown)

$\tilde{W}_{i \rightarrow j}$  : reconstructed from margins  $\left\{ \begin{array}{l} s_i^{in} = \sum_{j \in V} w_{j \rightarrow i} \\ s_i^{out} = \sum_{j \in V} w_{i \rightarrow j} \end{array} \right.$

## Traditional approach

(from “strengths” only)

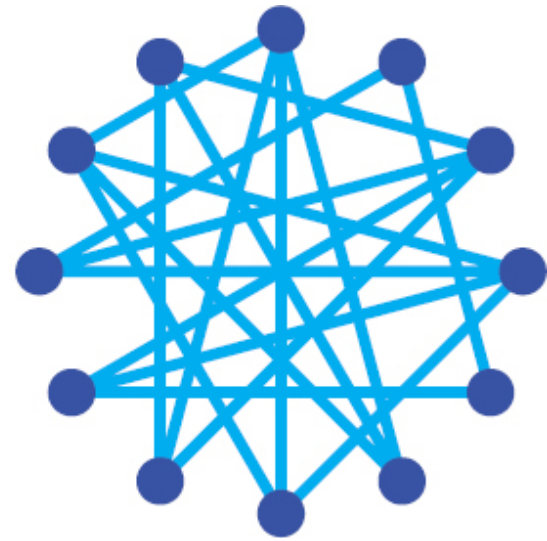


$$\tilde{W}_{i \rightarrow j} = \frac{s_i^{out} s_j^{in}}{W}$$

margins: **OK**, topology: **BAD**

## Enhanced method

(from strengths + degrees)



$$\tilde{W}_{i \rightarrow j} = \frac{z^{-1} + s_i^{out} s_j^{in}}{W} \tilde{a}_{i \rightarrow j}$$

margins: **OK**, topology: **OK**

# Part 3:

if systems are at (quasi-)equilibrium,  
their structure can be  
**modeled with  
explanatory variables**

(which should couple to the  
“right” constraints)



# Same story for international trade

- Jan Tinbergen: 1<sup>st</sup> Nobel Memorial Prize in Economics, 1969
- Leiden, 1929: PhD Thesis in physics  
“Minimumproblemen in de natuurkunde en economie” (supervisor P. Ehrenfest)

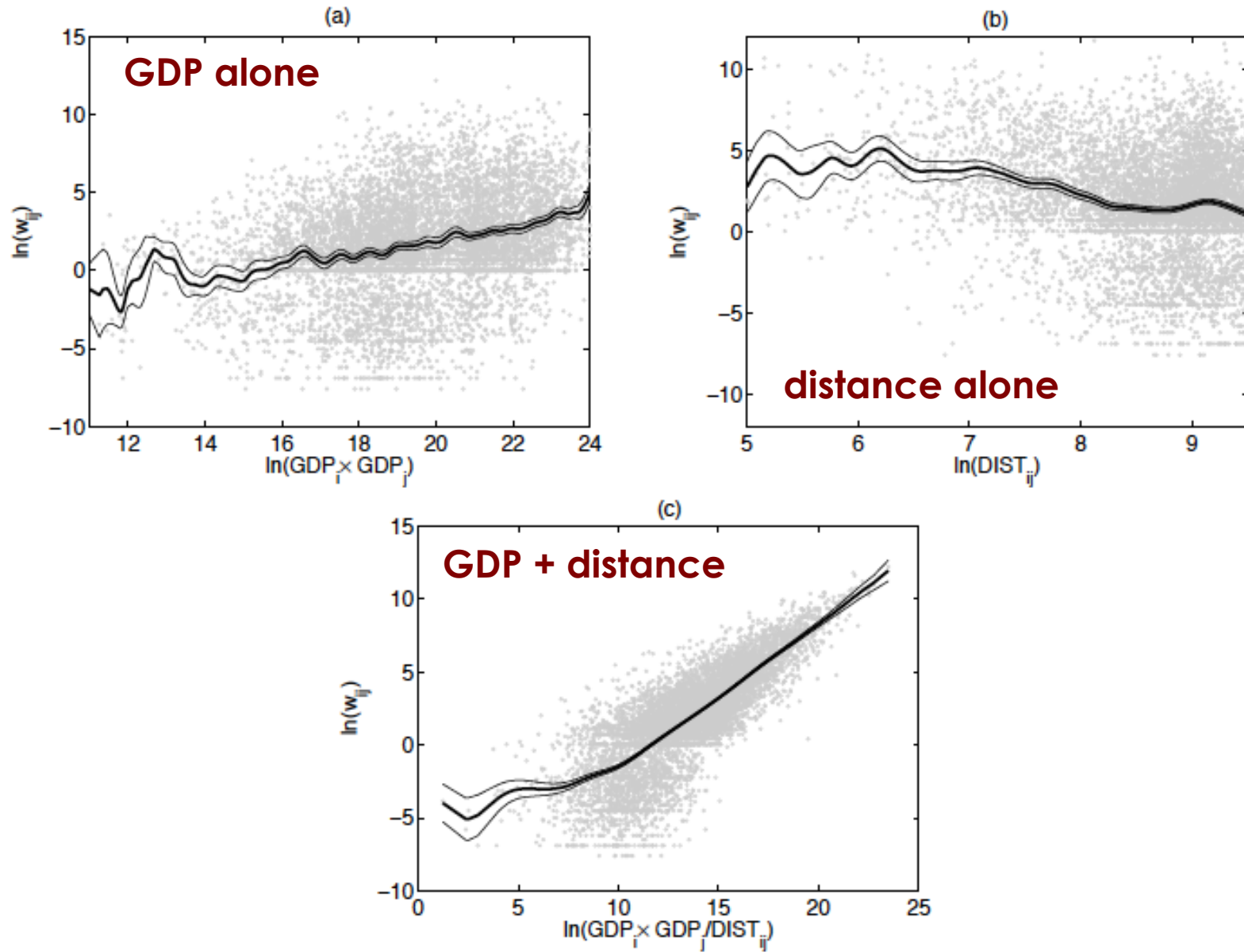
The (simple) **Gravity Model** of international trade:

$$\langle w_{ij} \rangle = \alpha \cdot GDP_i^\beta \cdot GDP_j^\beta \cdot D_{ij}^\gamma \cdot X_{ij}^\epsilon$$

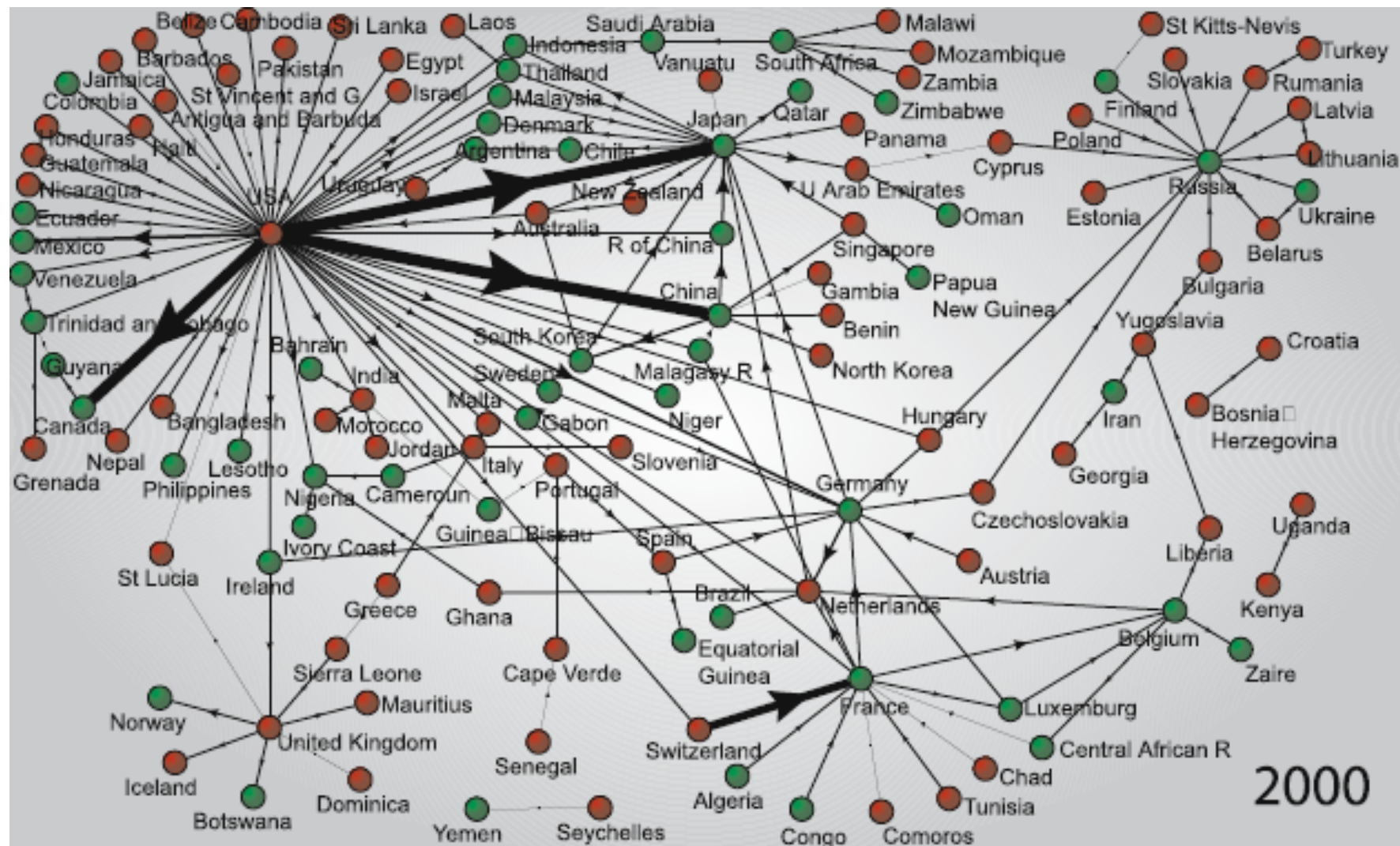
Simplest case:  $\beta \approx -\gamma \approx 1$ ,  $\epsilon \approx 0$  (as in Newton's law)

J. Tinbergen, *Shaping the World Economy: suggestions for an international economic policy* (the Twentieth Century Found, New York, 1962).

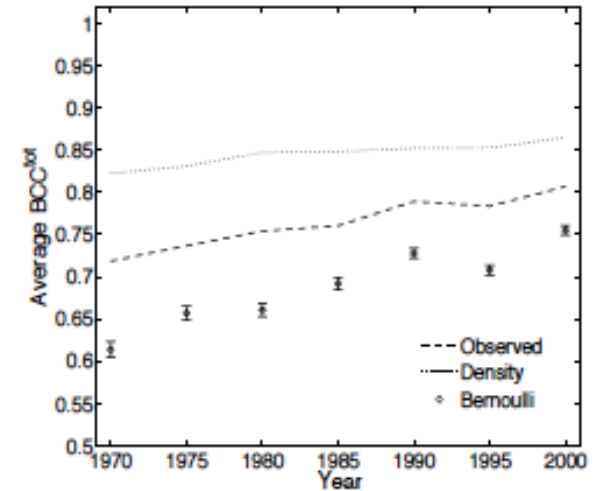
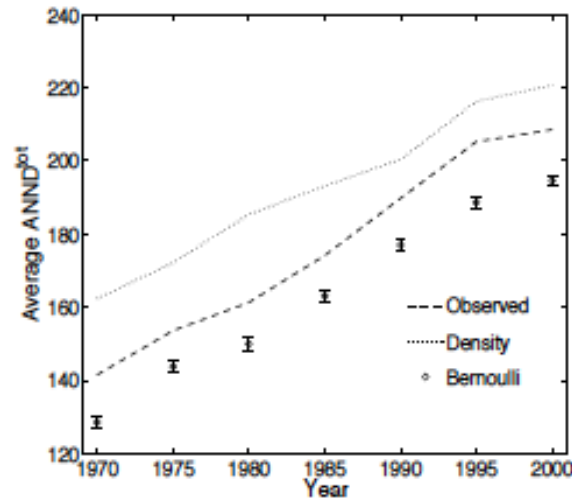
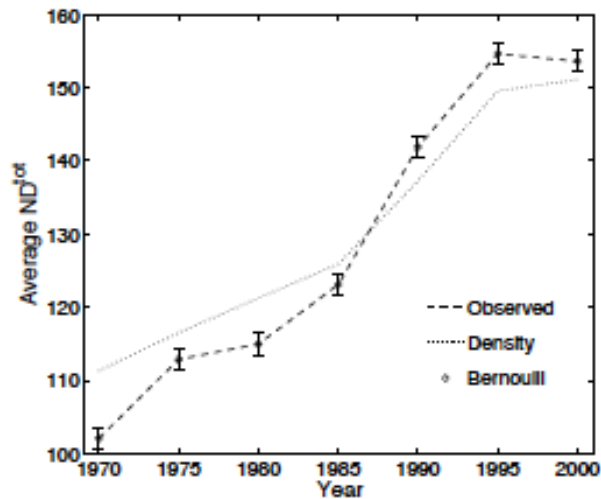
# The Gravity Model works well for “non-zeroes”



... **but:** the International Trade Network (ITN) has a complex topology!



# 'Against gravity'



Duenas & Fagiolo, LEM Working Paper, Scuola Superiore Sant'Anna, Pisa (2011)

Even if only the **correct number of links** (*left panel*) is placed where the “gravity” is stronger, the (density-induced) GM predicts **too much assortativity** (*center*) and **clustering** (*right*)

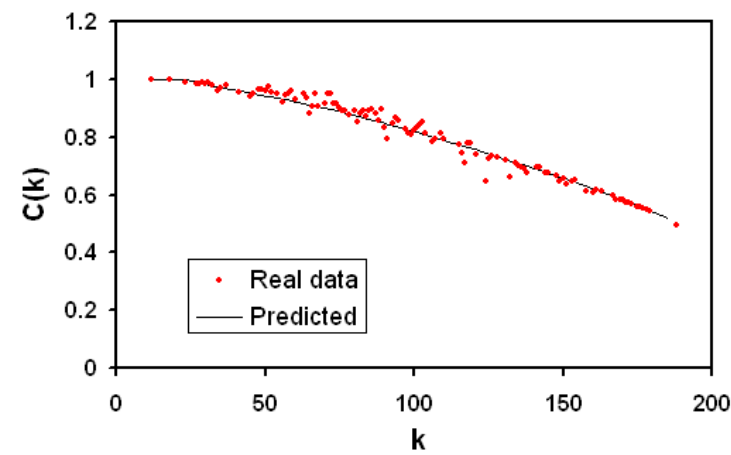
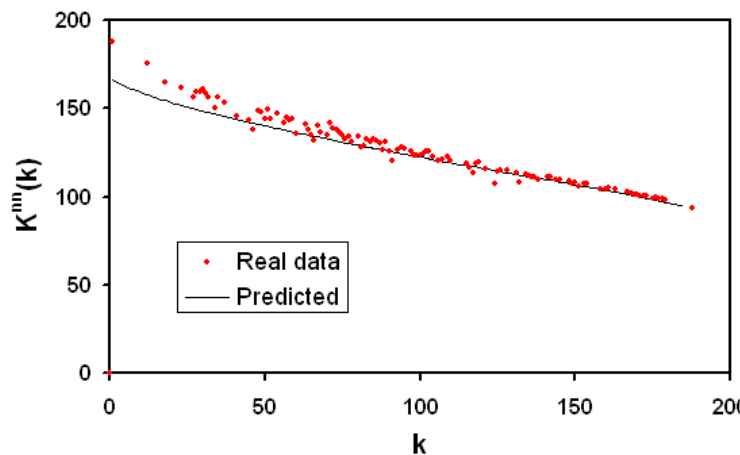
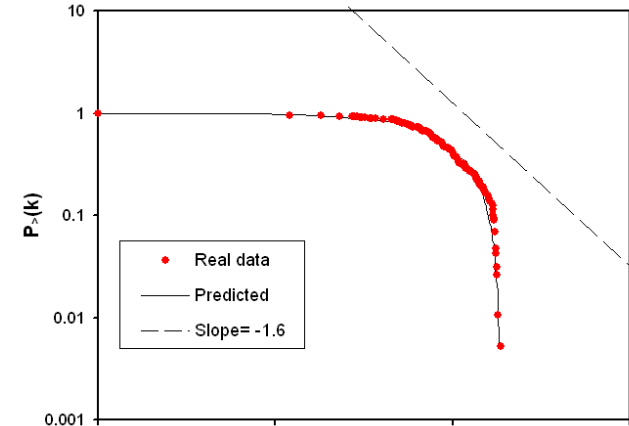
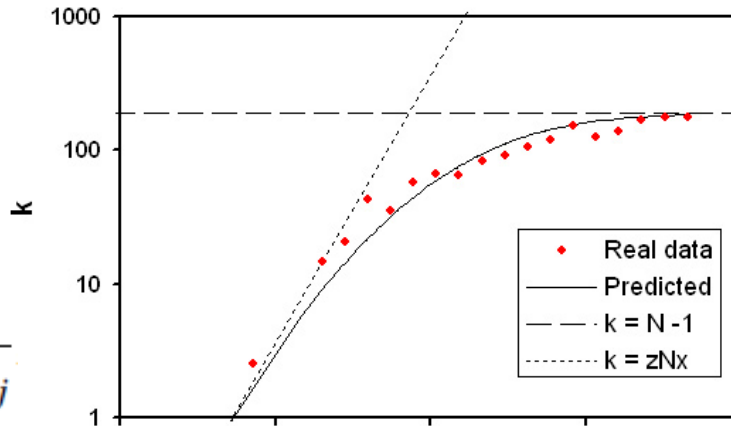
=> 'repulsion' where 'attraction' is expected, and vice versa!

# GDP-driven model of Trade Network

Replacing hidden variables with country GDP in the **binary configuration model** yields the Fitness Model (Caldarelli et al. PRL 2002)

$$f(x_i, x_j) = \frac{\delta x_i x_j}{1 + \delta x_i x_j}$$

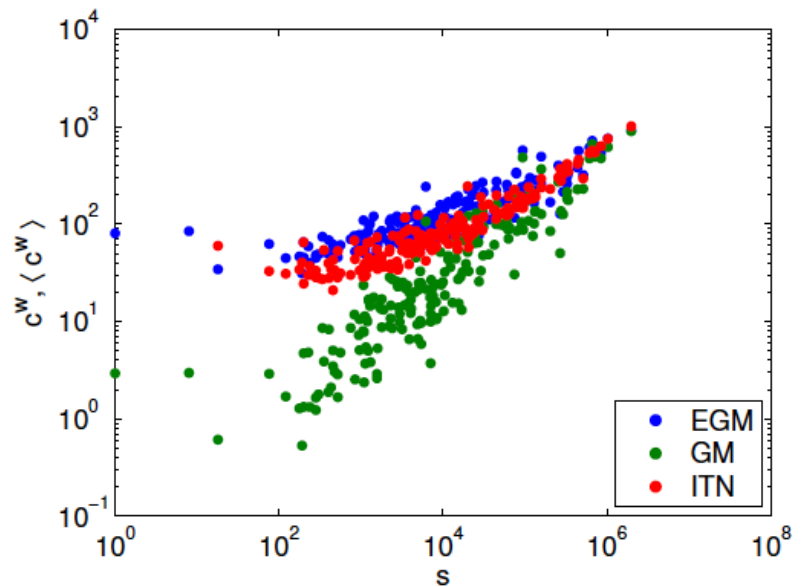
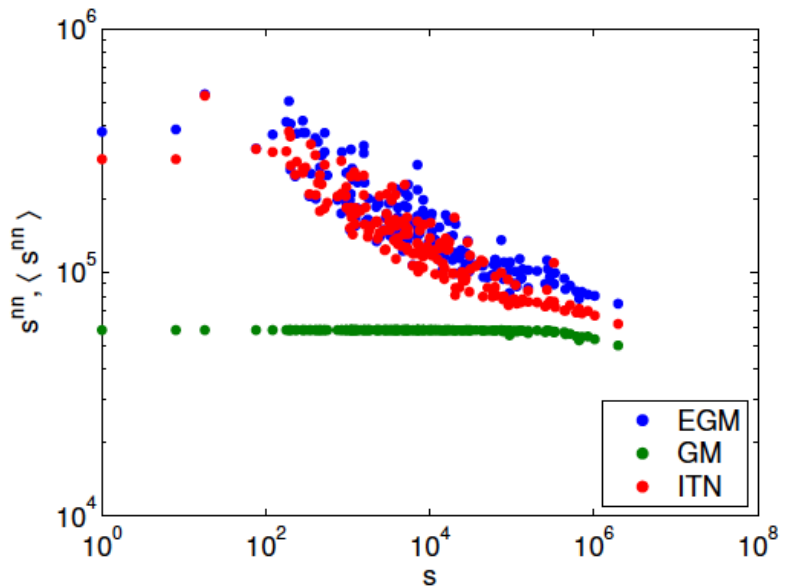
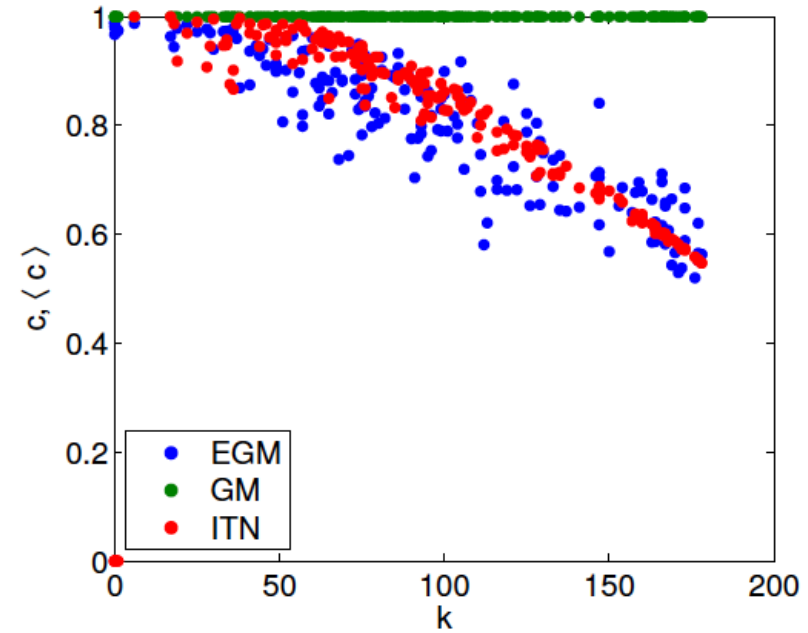
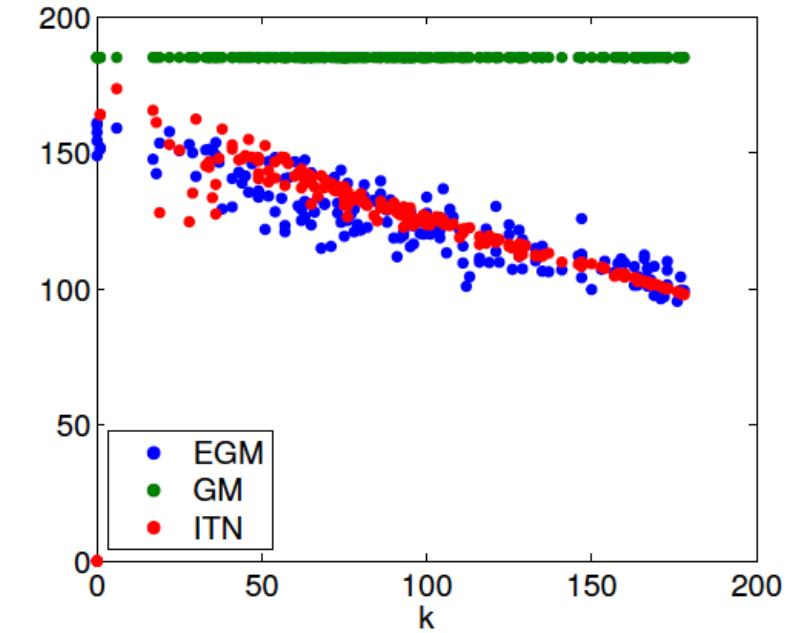
$$\tilde{L} = \frac{1}{2} \sum_{i=1}^N \sum_{j \neq i} f(x_i, x_j)$$



- Garlaschelli, Loffredo *Physical Review Letters* 93, 188701 (2004)
- Garlaschelli, Loffredo *Physica A* 355, 138 (2005)
- Garlaschelli, Loffredo *Physical Review E* 78, 015101(R) (2008)
- Garlaschelli, Di Matteo, Aste, Caldarelli, Loffredo *European Physical Journal B* 57, 159 (2007)

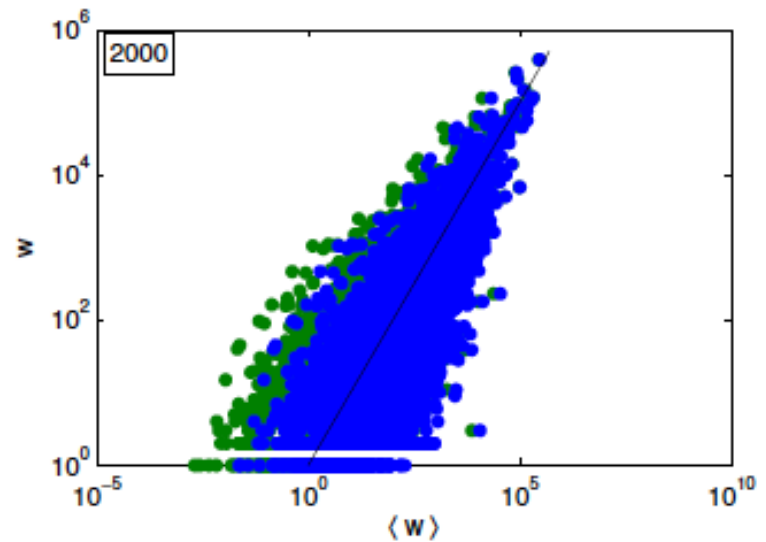
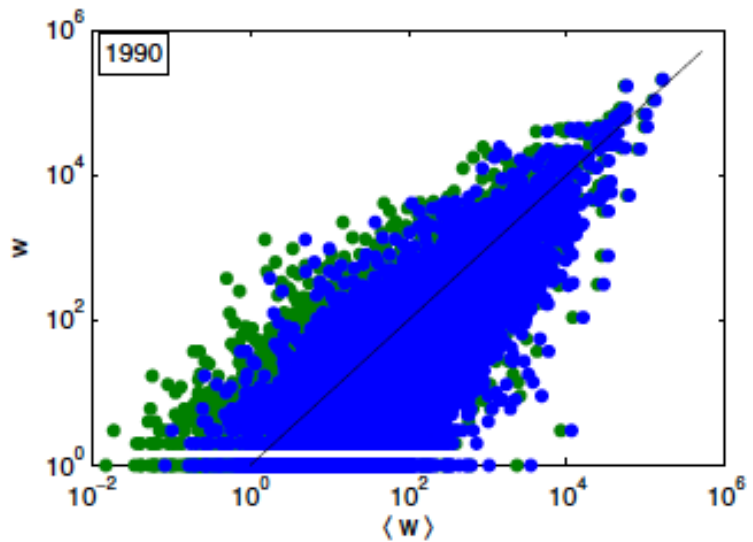
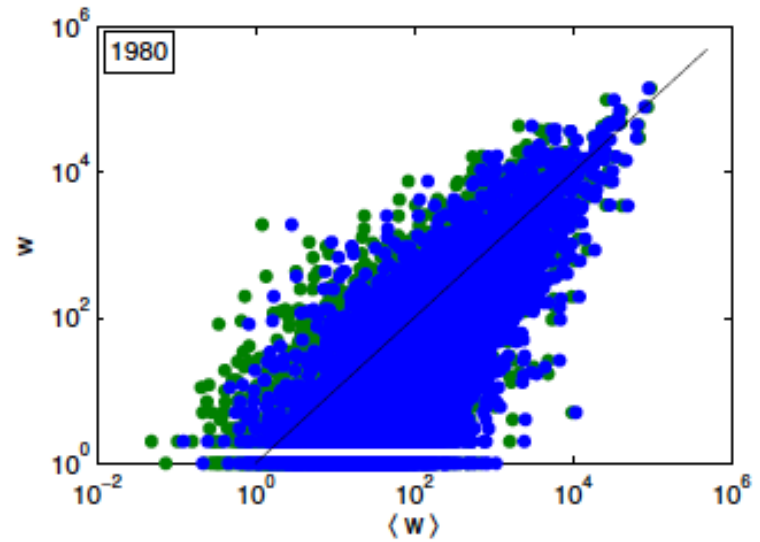
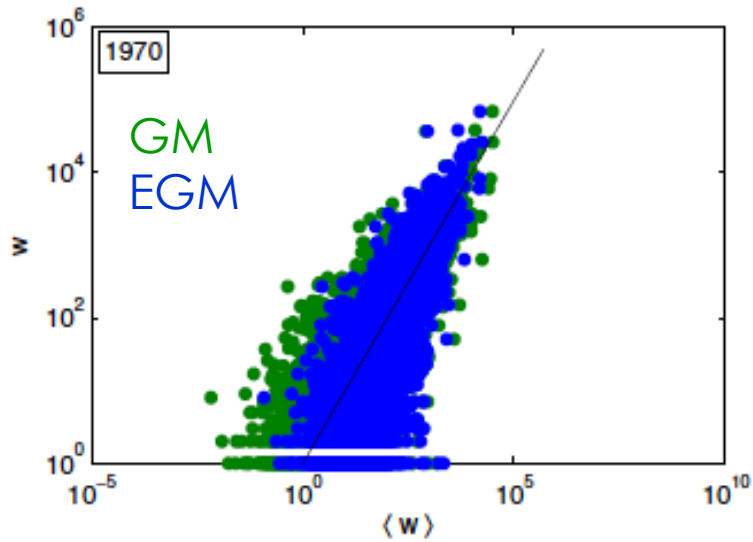


# Adding weights: Enhanced Gravity Model



Almog, Bird, Garlaschelli, <http://arxiv.org/abs/1506.00348> (2015).

# Adding weights: **Enhanced Gravity Model**

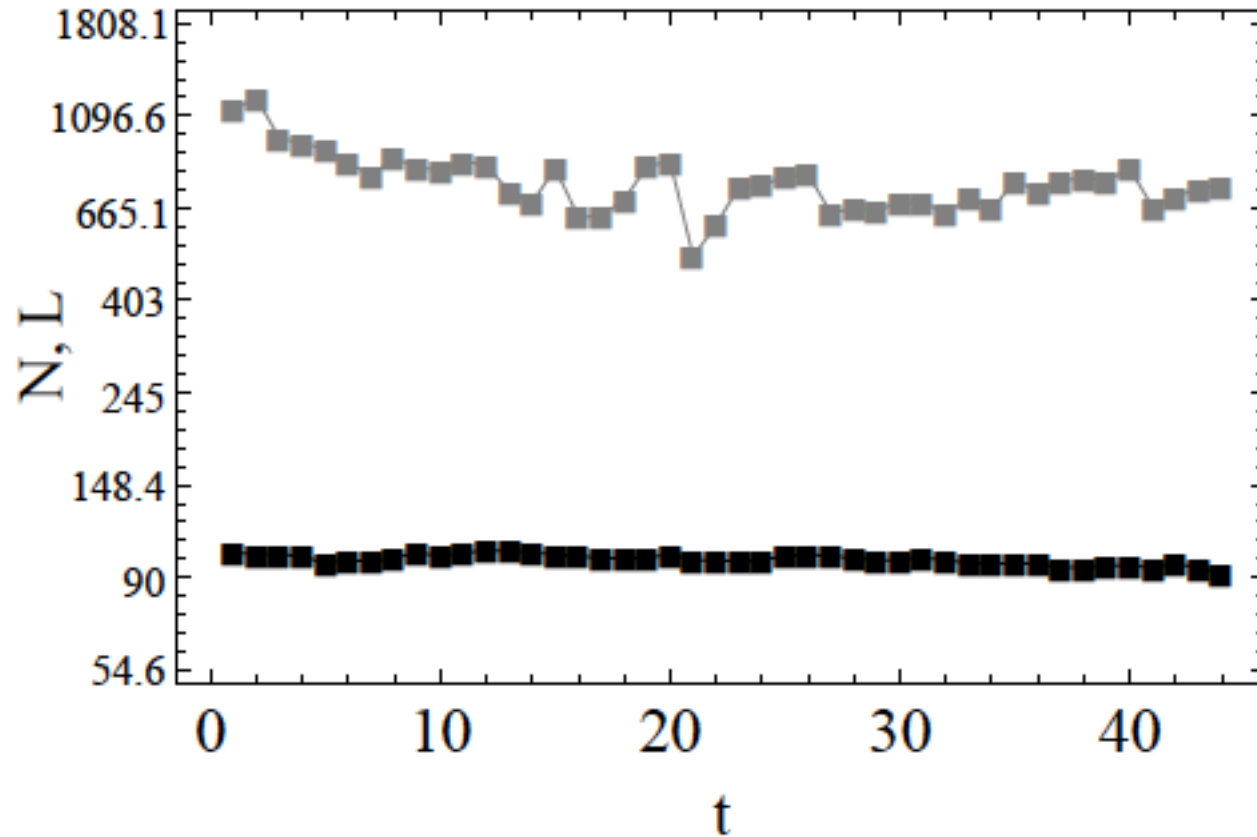


# Part 4:

if systems are **out of equilibrium**,  
reconstruction and modelling  
are **unreliable...**

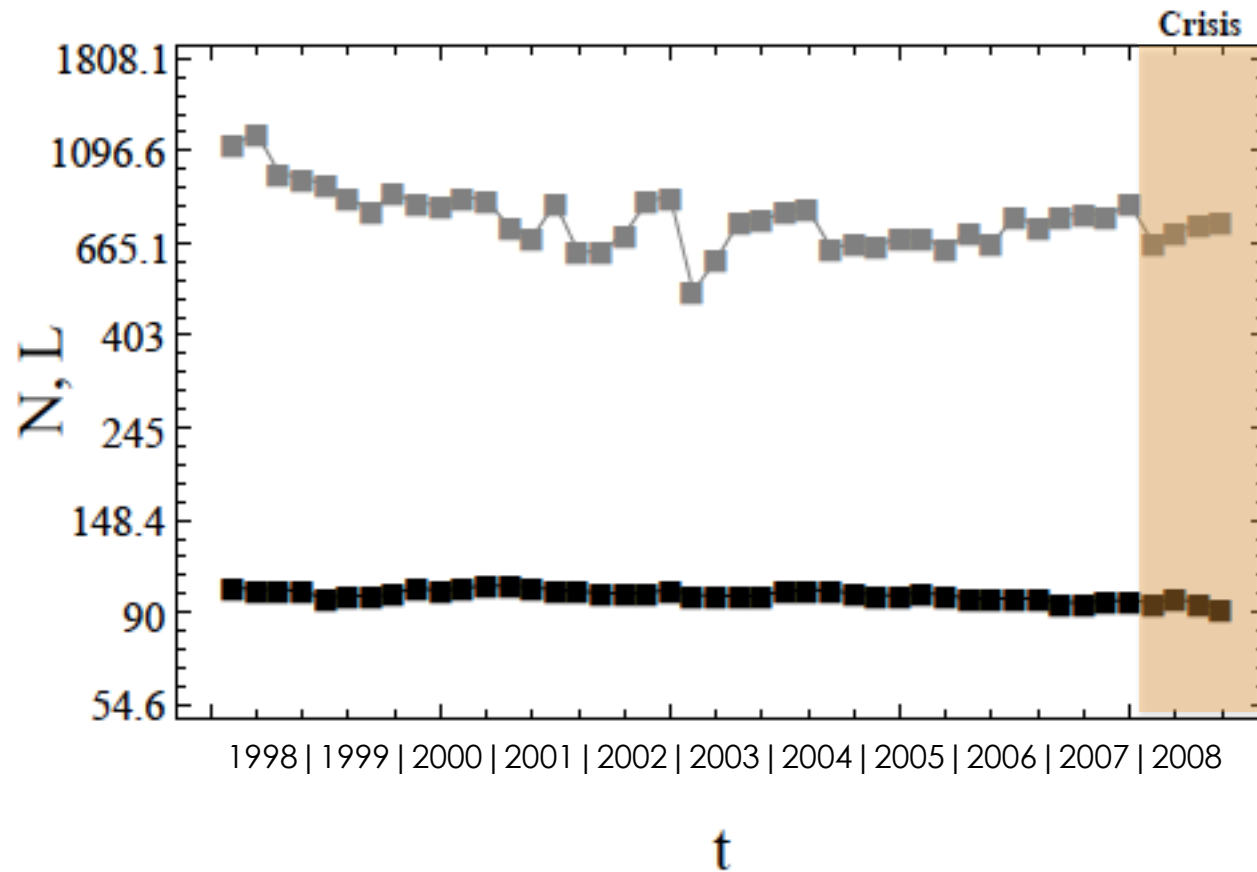
...but may still be crucial to build  
**early-warning signals**

# Dutch banks: signs of the crisis?



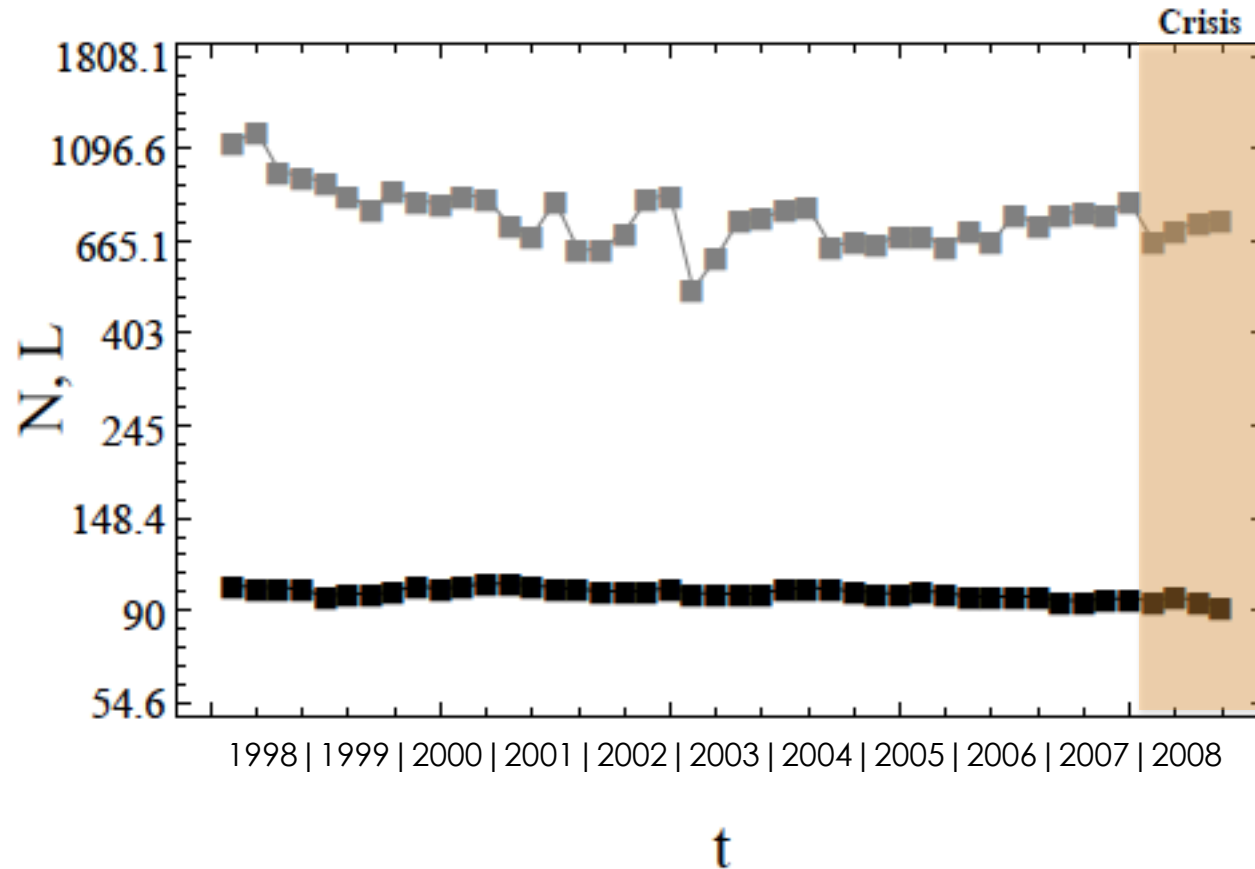
Size & density

# Dutch banks: signs of the crisis?



Size & density

# Dutch banks: signs of the crisis?



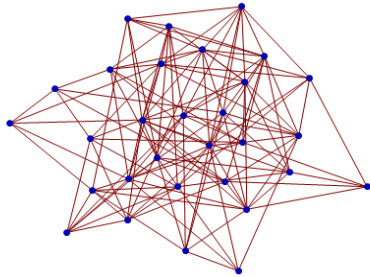
Size & density

No sign of crisis?

Maybe not visible from purely topological quantities?

# Homogeneous benchmark/null model

Controlling for overall size and density of the network  
(random graph)



**Constraint:**  
number of links  
(and nodes)

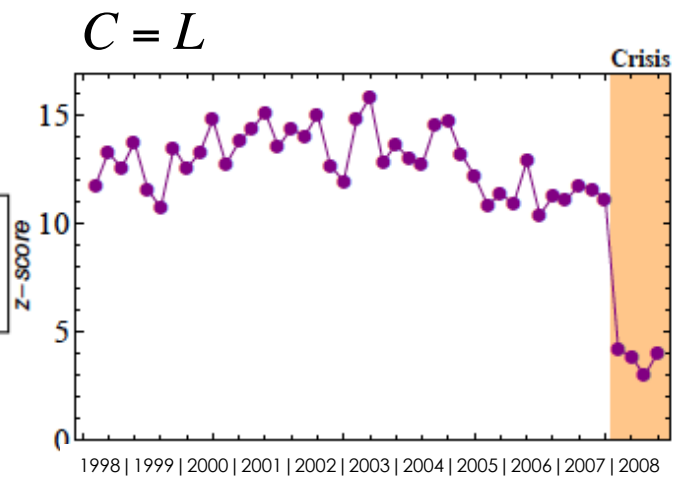
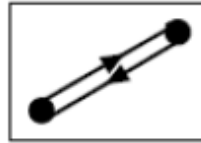
$$C = L$$

Comparing observed ( $X$ ) and randomized ( $\langle X \rangle$ ) properties:

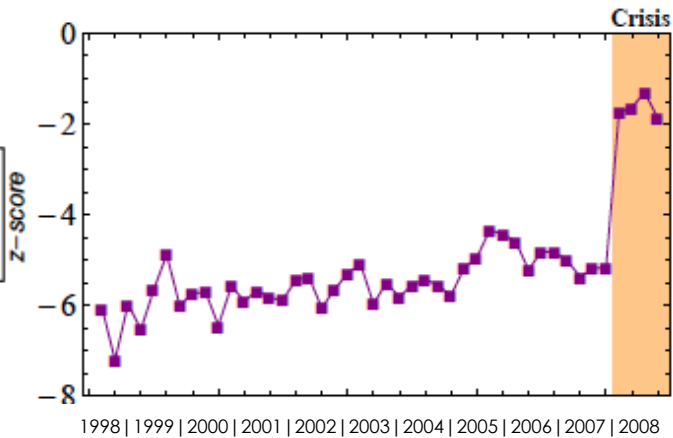
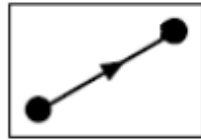
**z-score** 
$$z_X \equiv \frac{X - \langle X \rangle}{\sigma[X]}$$

# Seeing the crisis?

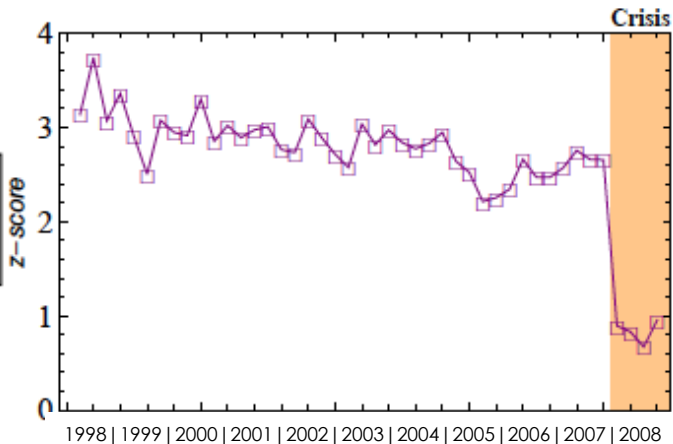
Full dyads  
(reciprocated)



Single dyads  
(non-reciprocated)



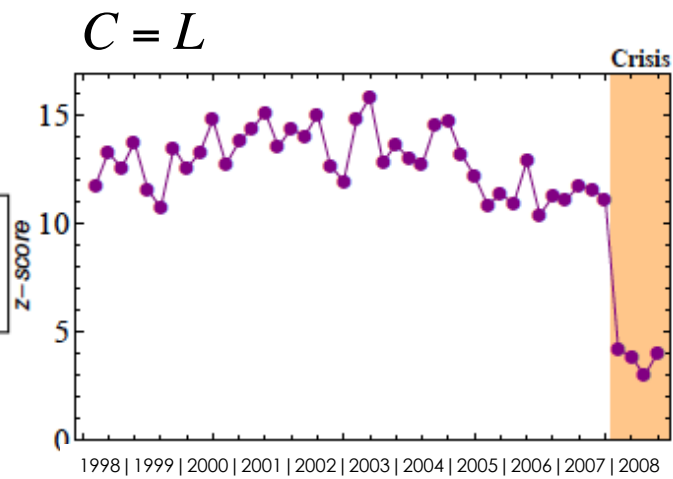
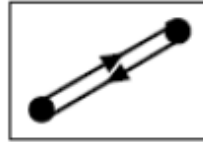
Empty dyads  
(disconnected)



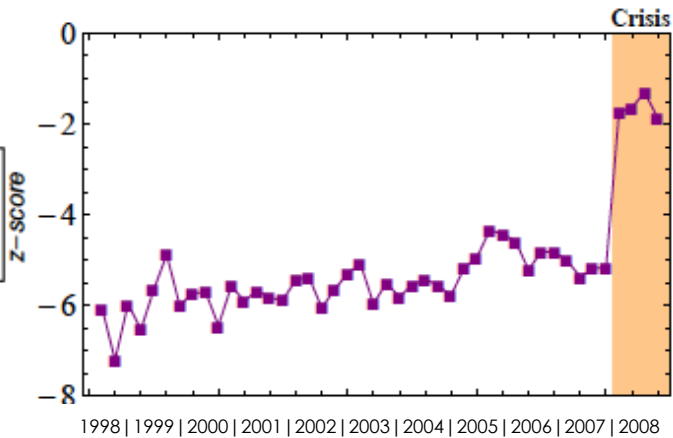
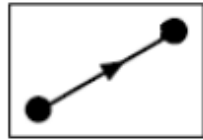


# Seeing the crisis?

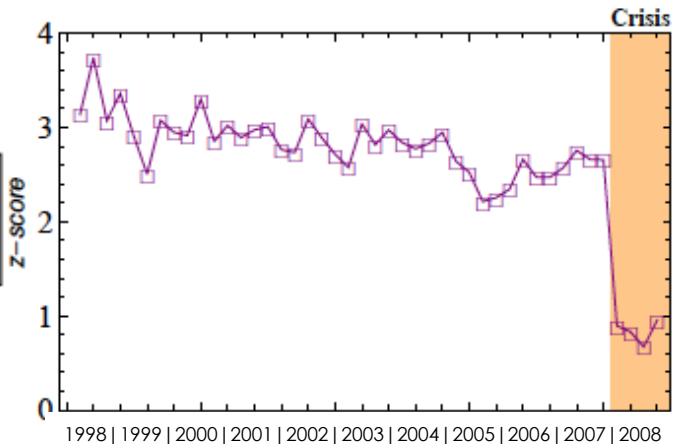
Full dyads  
(reciprocated)



Single dyads  
(non-reciprocated)



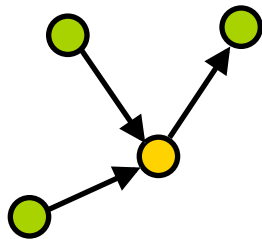
Empty dyads  
(disconnected)



Sudden 'dyadic collapse' to random graph

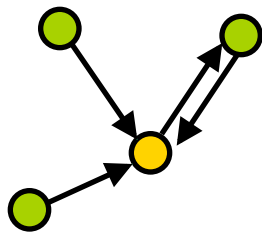
# Heterogeneous benchmark

Controlling for different connectivities of banks:  
**heterogeneous** benchmark/null model



**Constraints:**  
in-degree &  
out-degree  
(of each node)

$$\vec{C} = \{k_i^{in}, k_i^{out}\}$$



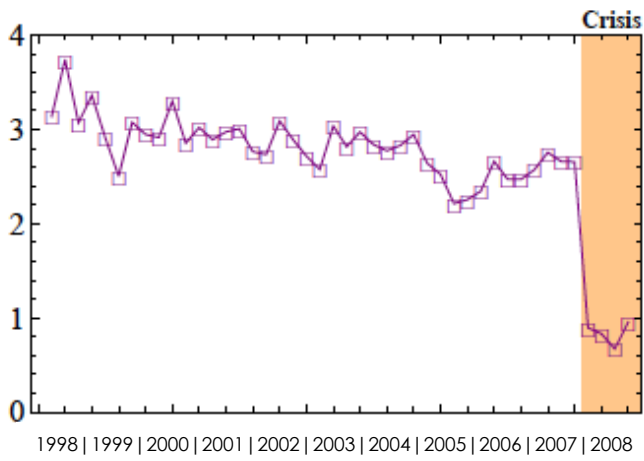
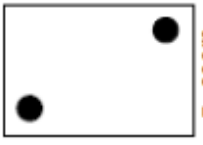
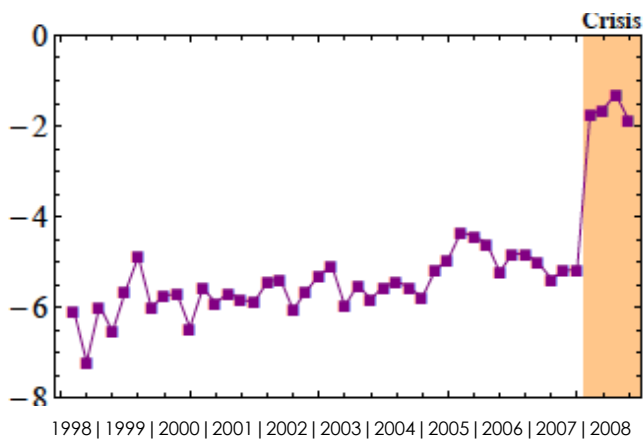
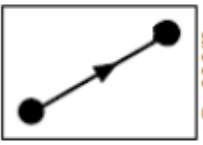
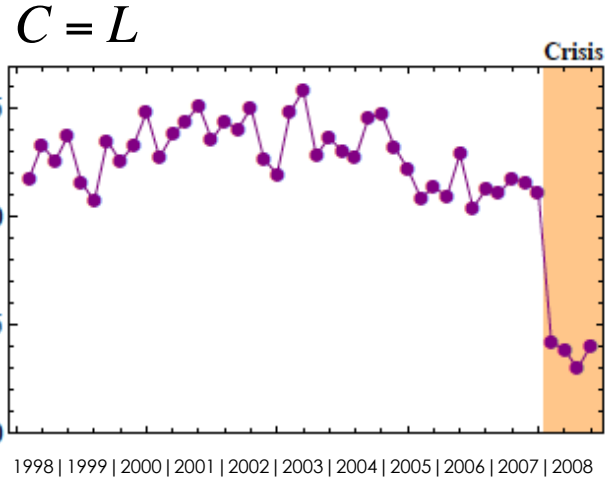
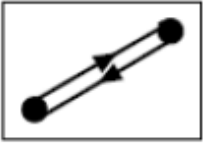
**Constraints:**  
in-degree,  
out-degree &  
reciprocal degree

$$\vec{C} = \{k_i^{in}, k_i^{out}, k_i^{both}\}$$

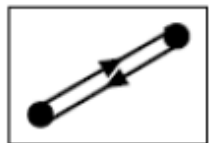
**z-score**

$$z_X \equiv \frac{X - \langle X \rangle}{\sigma[X]}$$

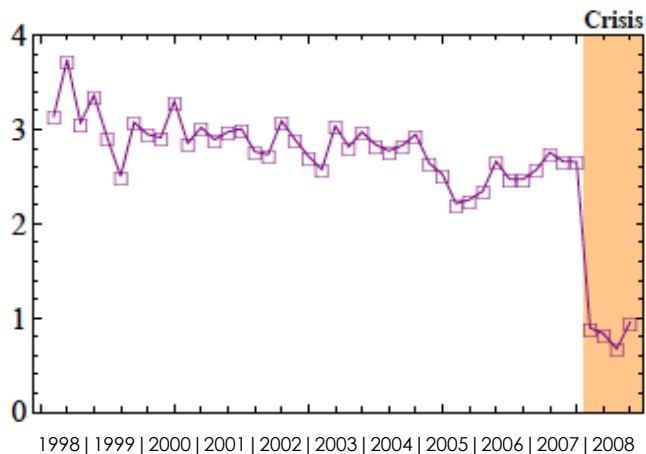
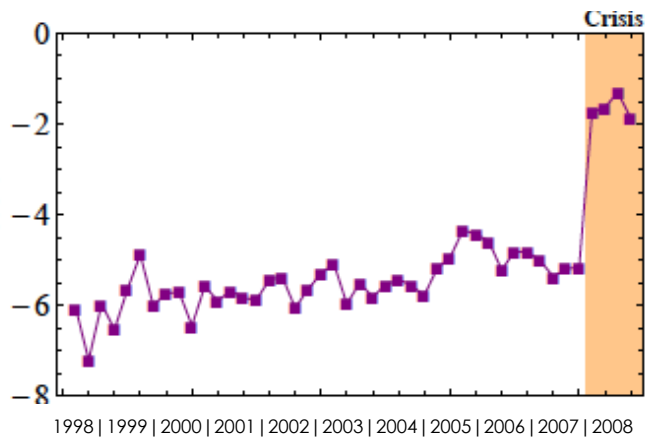
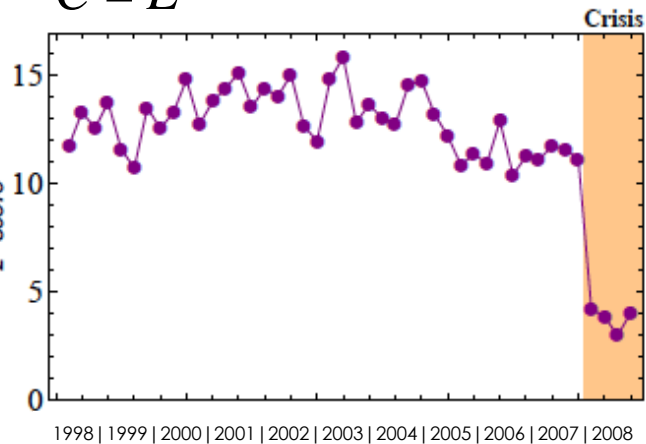
... seeing  
the crisis?



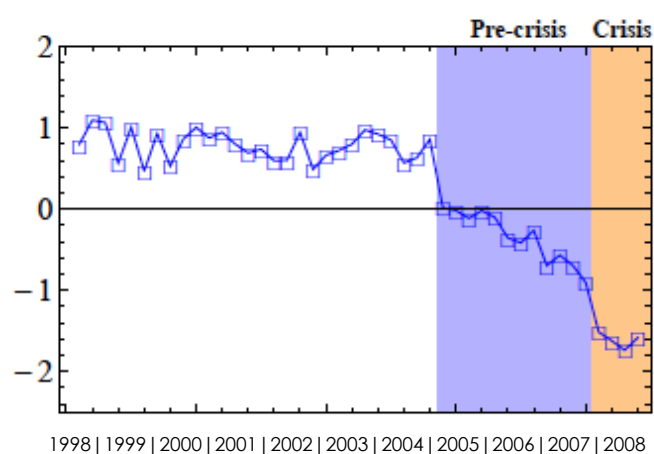
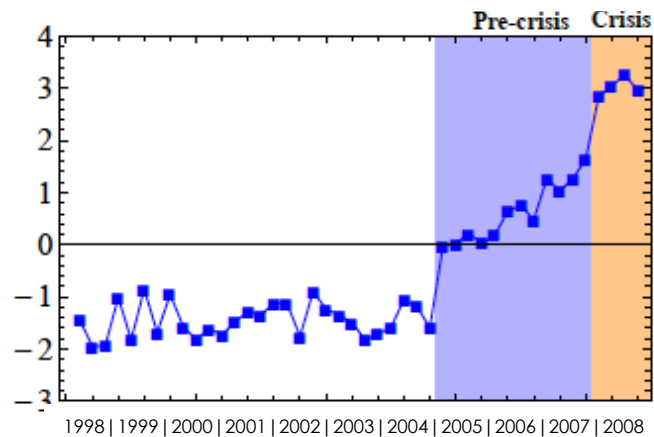
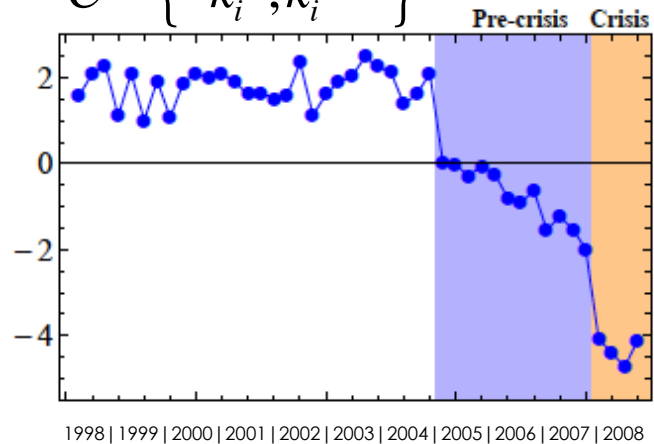
# Foreseeing the crisis?



$$C = L$$



$$\vec{C} = \{ k_i^{in}, k_i^{out} \}$$



# Heterogeneity (= “right” constraints) matters!

As seen from a **homogeneous** benchmark,  
the collapse appears sudden (**abrupt transition**)

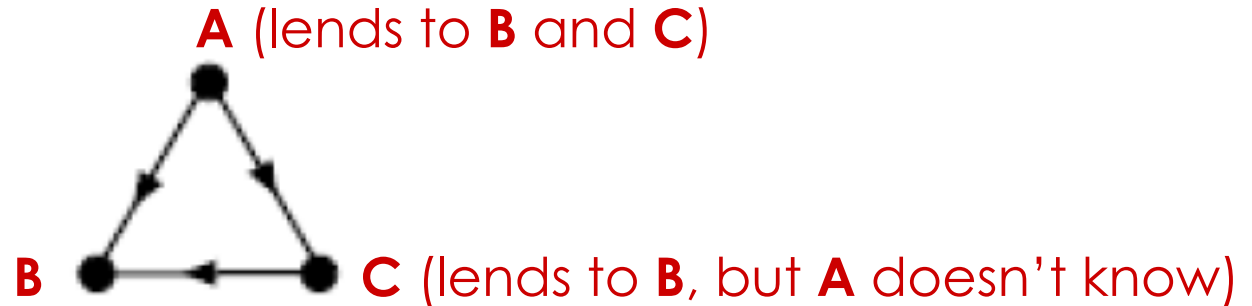
As seen from a **heterogeneous** benchmark,  
the collapse appears gradual (**continuous transition**)

Note: the **measured** quantities are the same in the  
two cases; what changes is their **expected** value!

$$z_X \equiv \frac{X - \langle X \rangle}{\sigma[X]}$$

# From dyads to triads

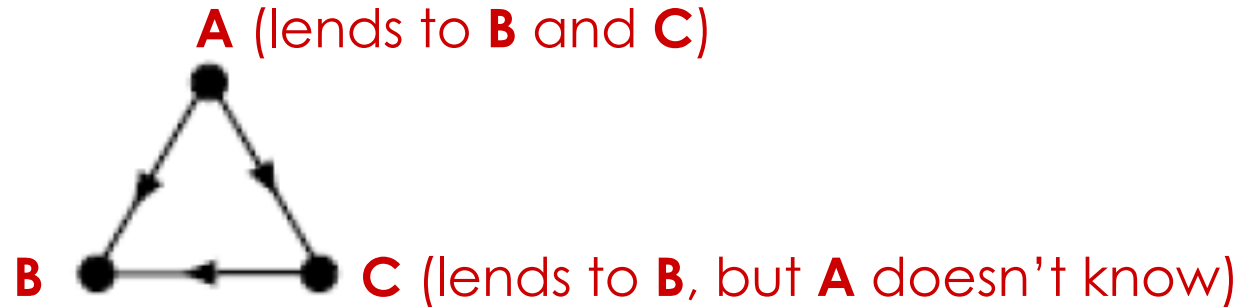
OTC markets: underestimation of **counterparty risk**



**A** is (hopefully) prepared to the direct effect of **B**'s default, but not to the indirect effects of **B**'s default through **C**.

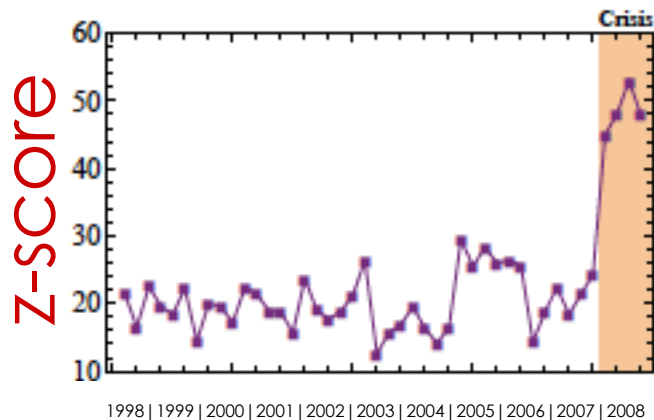
# From dyads to triads

OTC markets: underestimation of **counterparty risk**



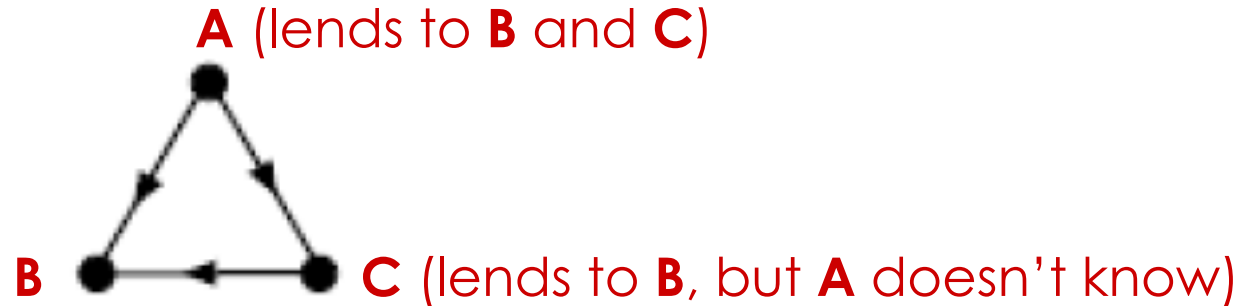
**A** is (hopefully) prepared to the direct effect of **B**'s default, but not to the indirect effects of **B**'s default through **C**.

$$C = L$$



# From dyads to triads

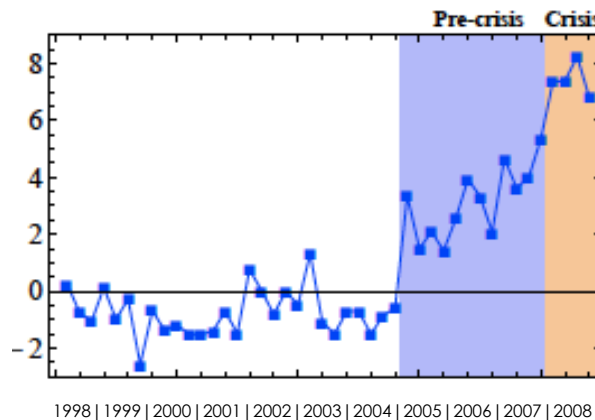
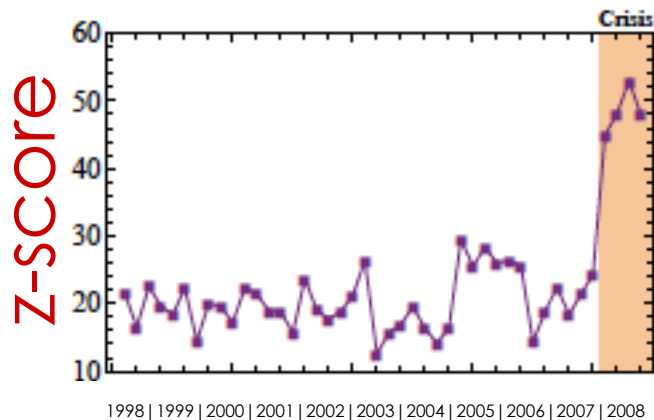
OTC markets: underestimation of **counterparty risk**



**A** is (hopefully) prepared to the direct effect of **B**'s default, but not to the indirect effects of **B**'s default through **C**.

$$C = L$$

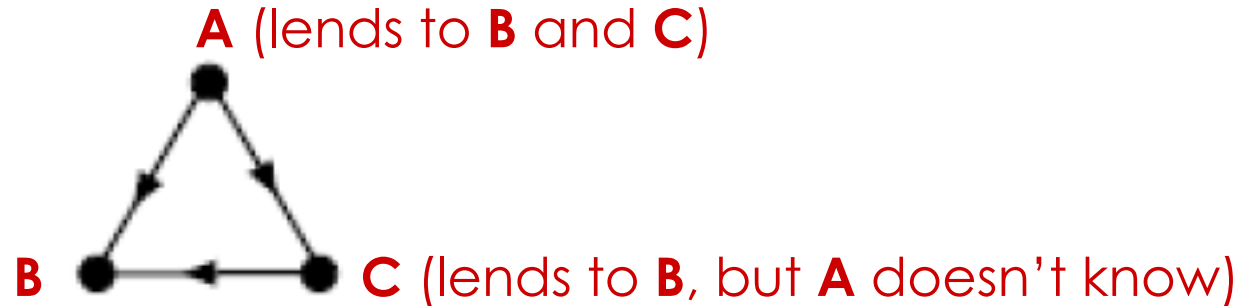
$$\vec{C} = \{k_i^{in}, k_i^{out}\}$$





# From dyads to triads

OTC markets: underestimation of **counterparty risk**

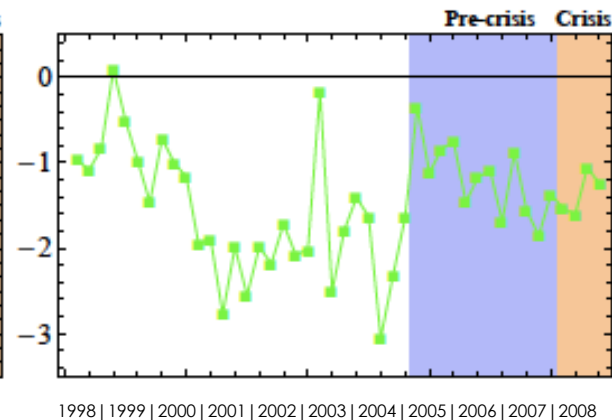
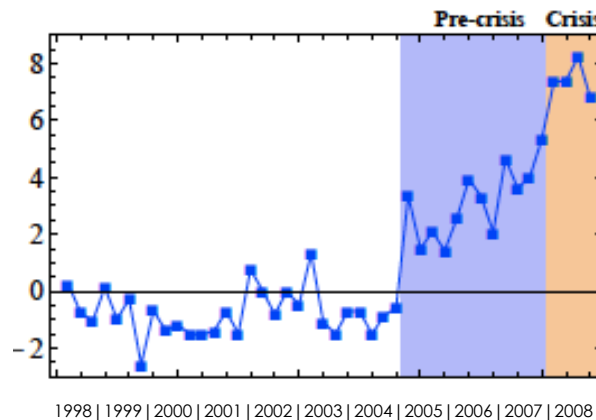
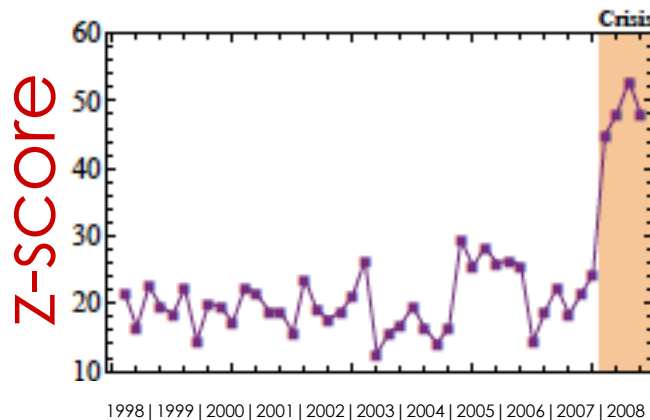


**A** is (hopefully) prepared to the direct effect of **B**'s default, but not to the indirect effects of **B**'s default through **C**.

$$C = L$$

$$\vec{C} = \{k_i^{in}, k_i^{out}\}$$

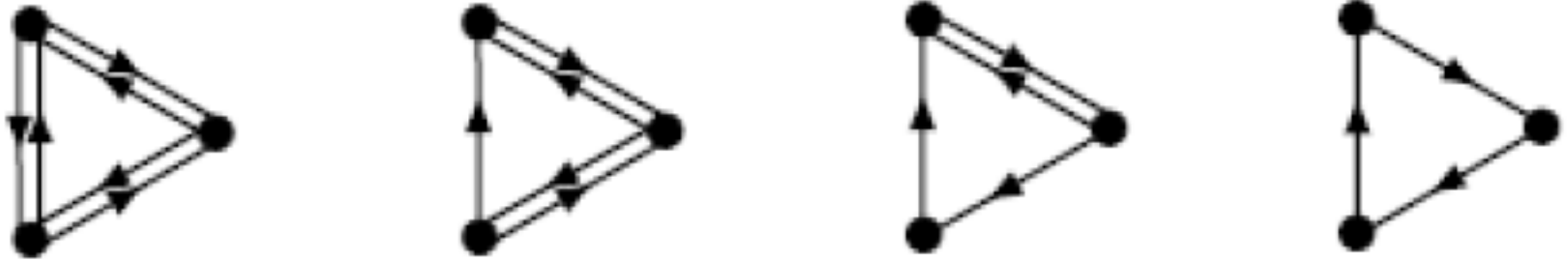
$$\vec{C} = \{k_i^{in}, k_i^{out}, k_i^{both}\}$$



(for triads, we need to filter out dyadic effects)

# Debt loops: 'risk autocatalysis'?

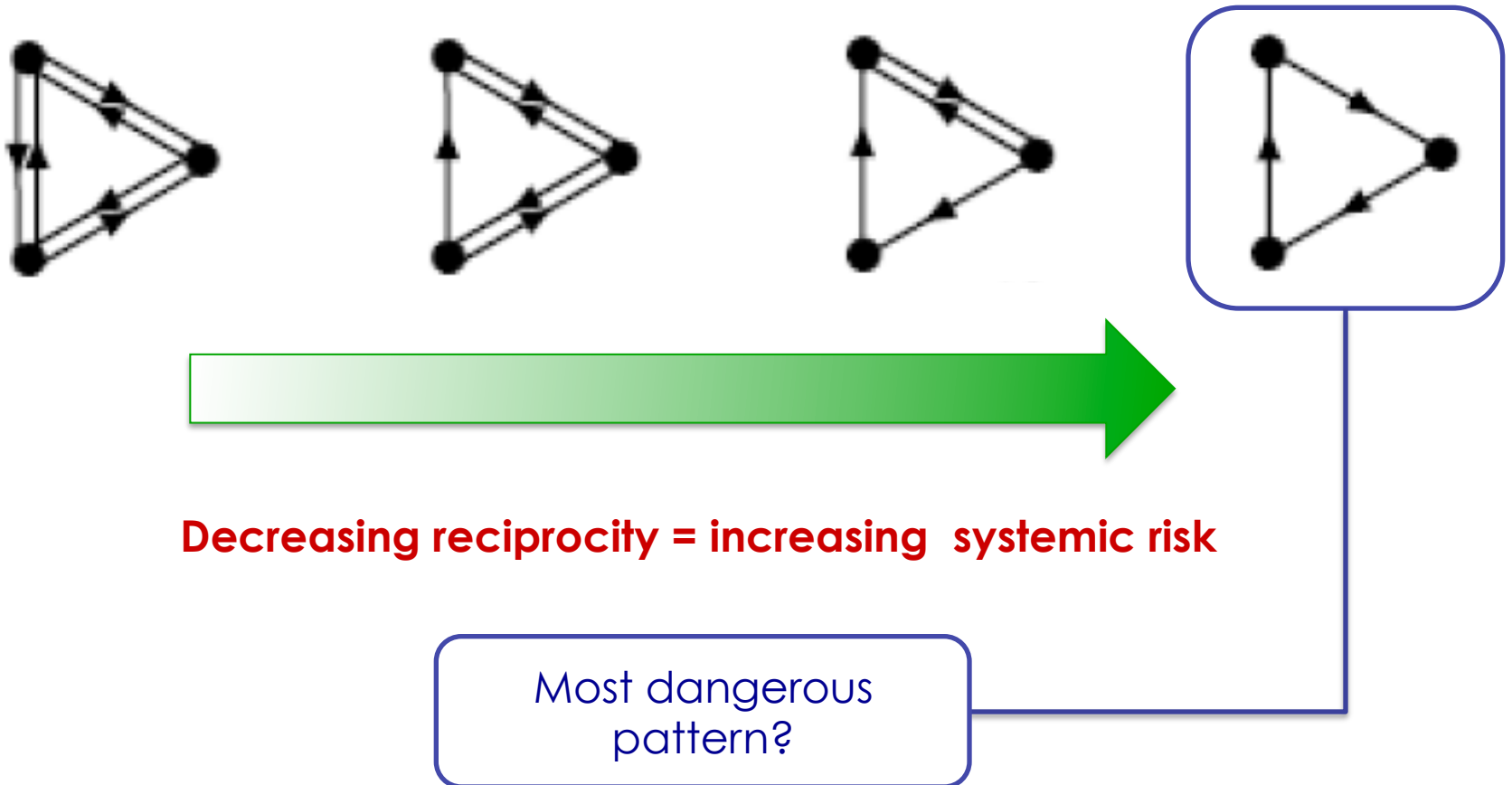
Circular lending loops:  
increased dependencies among default probabilities



**Decreasing reciprocity = increasing systemic risk**

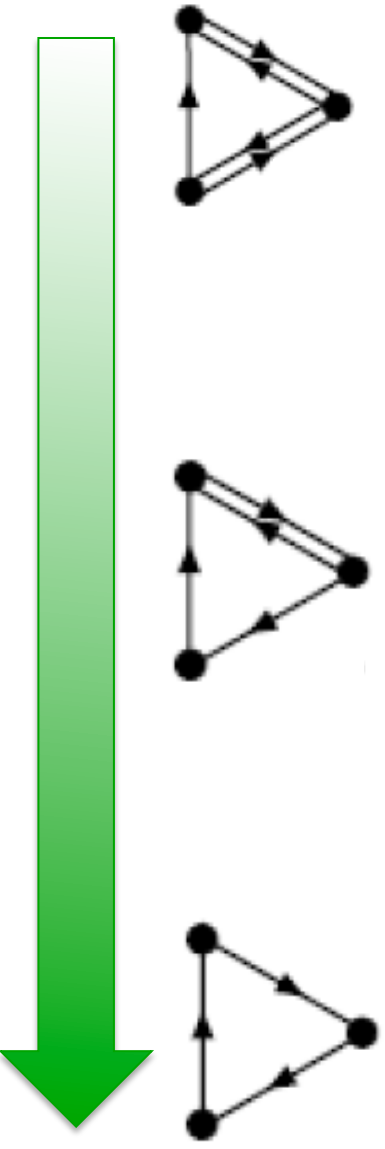
# Debt loops: 'risk autocatalysis'?

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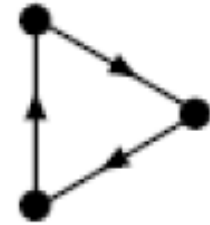
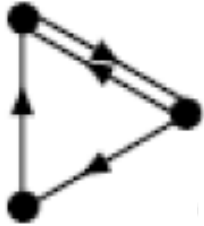
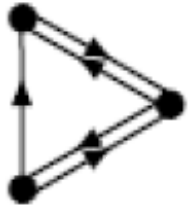
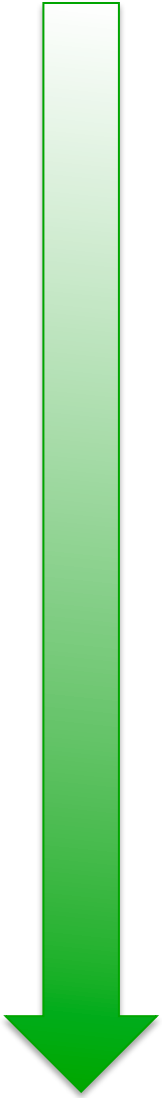
# Debt loops

**Decreasing reciprocity = increasing systemic risk**

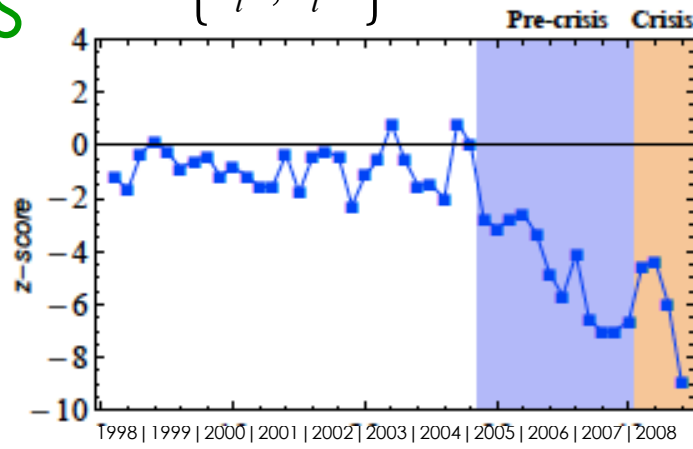


# Debt loops

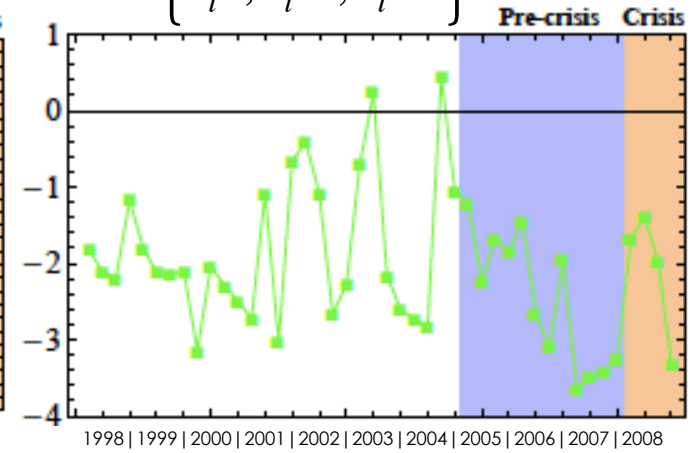
Decreasing reciprocity = increasing systemic risk



$$\vec{C} = \{k_i^{in}, k_i^{out}\}$$

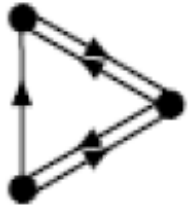
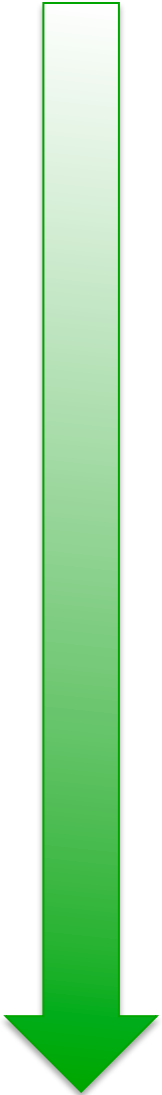


$$\vec{C} = \{k_i^{in}, k_i^{out}, k_i^{both}\}$$

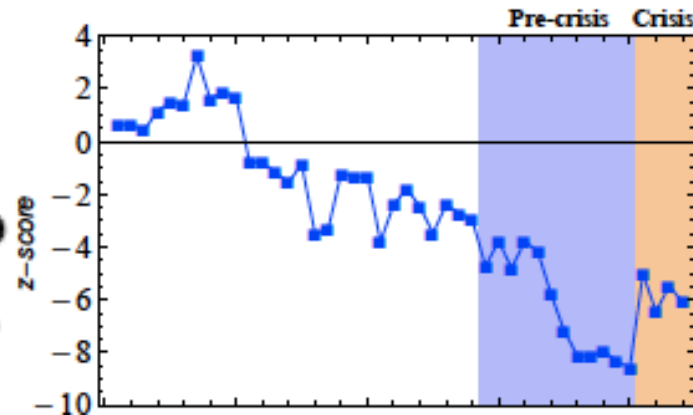
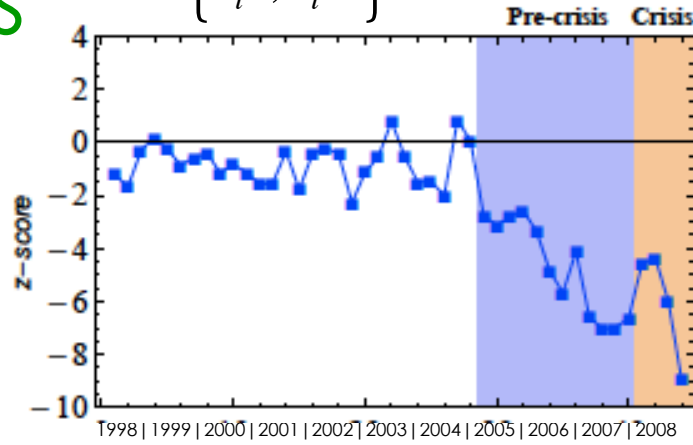


# Debt loops

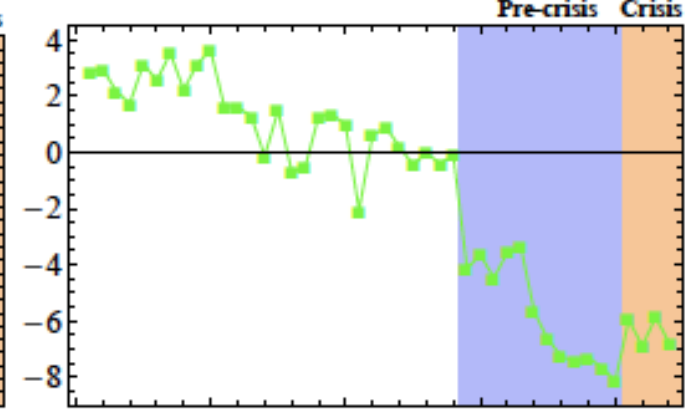
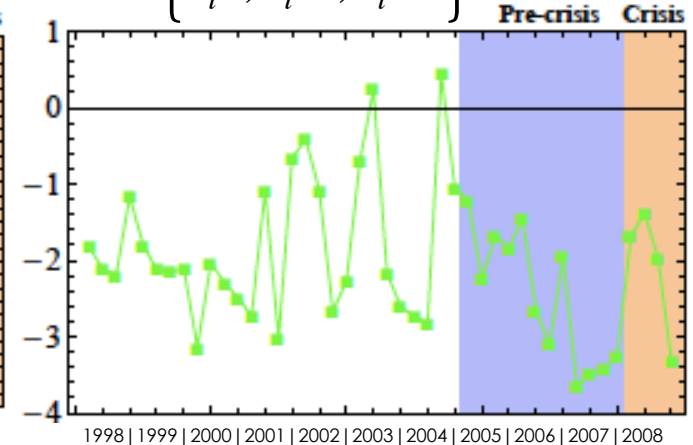
Decreasing reciprocity = increasing systemic risk



$$\vec{C} = \{k_i^{in}, k_i^{out}\}$$

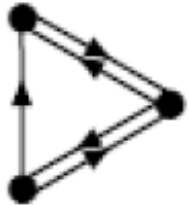
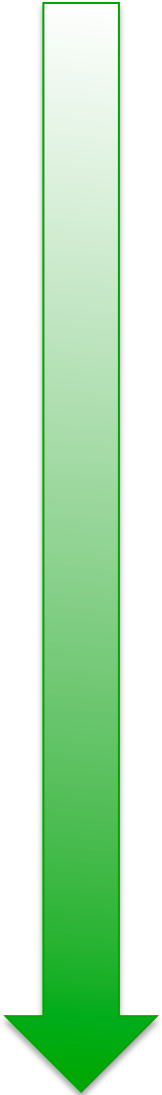


$$\vec{C} = \{k_i^{in}, k_i^{out}, k_i^{both}\}$$

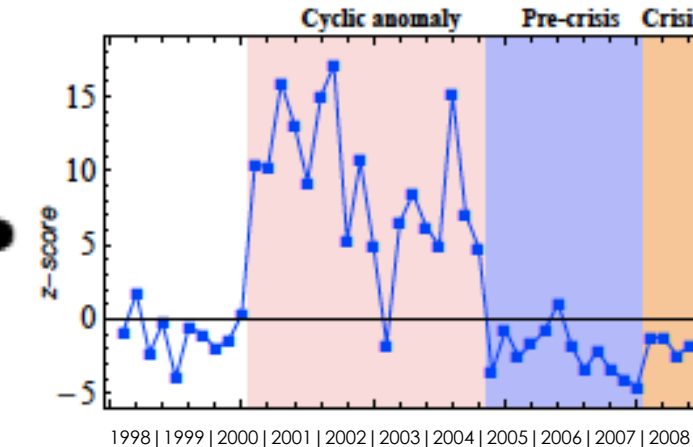
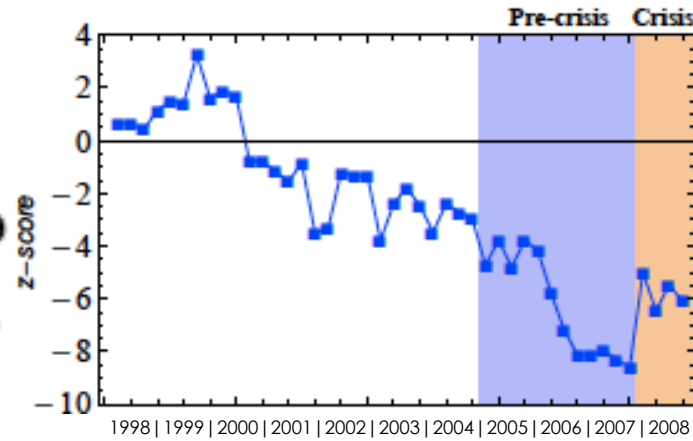
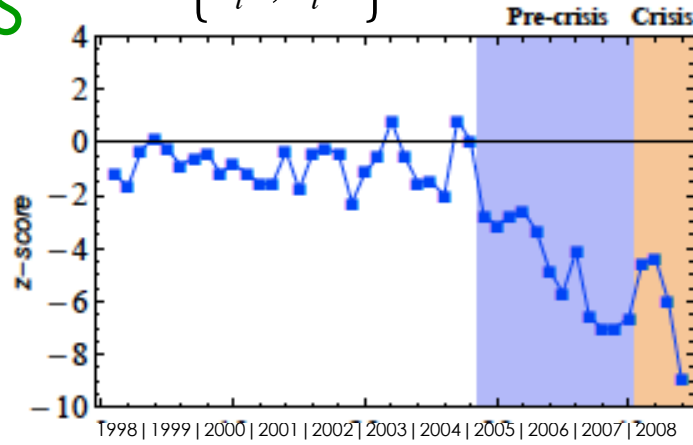


# Debt loops

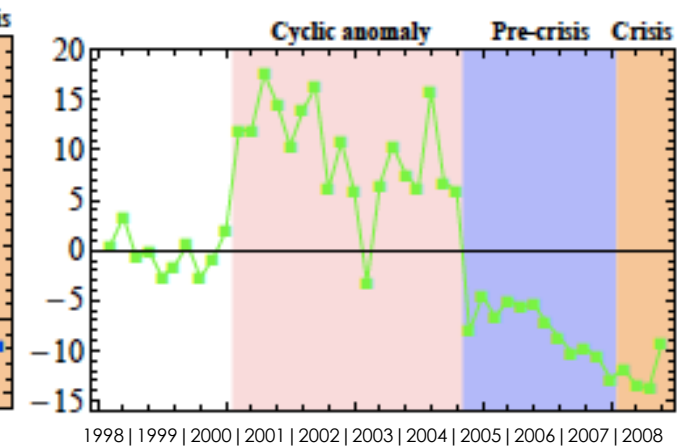
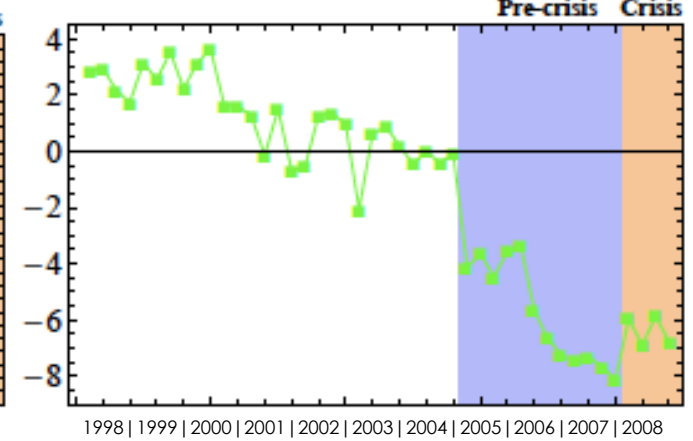
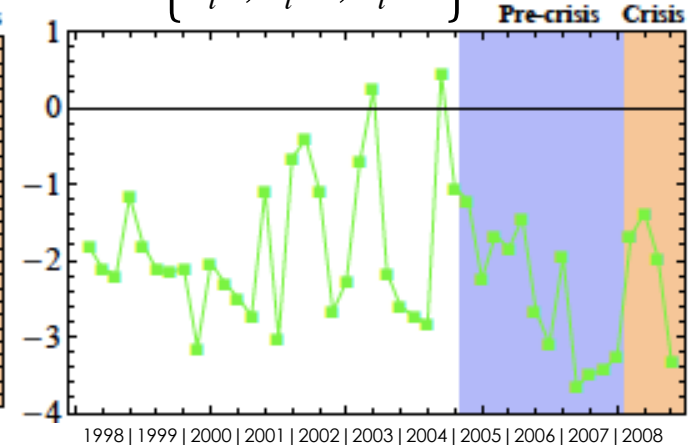
Decreasing reciprocity = increasing systemic risk



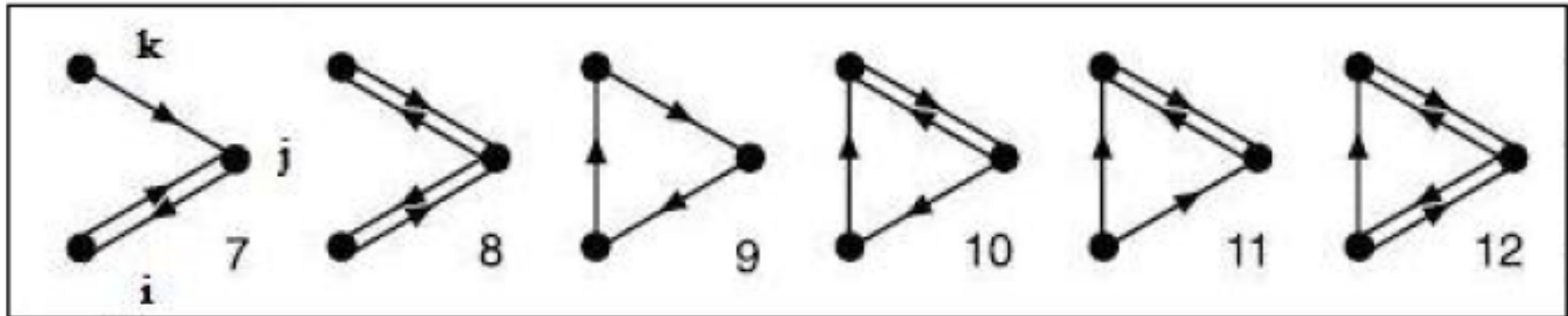
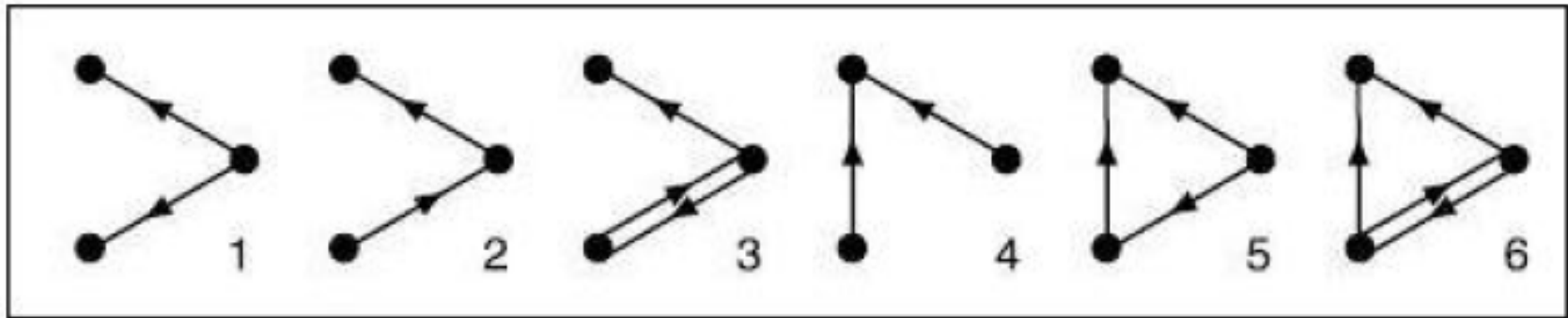
$$\vec{C} = \{k_i^{in}, k_i^{out}\}$$



$$\vec{C} = \{k_i^{in}, k_i^{out}, k_i^{both}\}$$

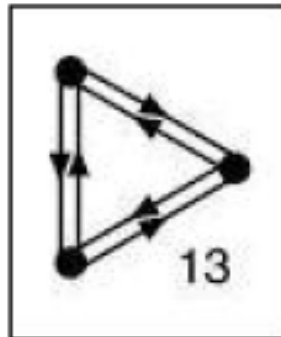


# ALL TRIADS



z-score:

$$z_m \equiv \frac{N_m(\mathbf{A}^*) - \langle N_m \rangle^*}{\sigma^*[N_m]}$$



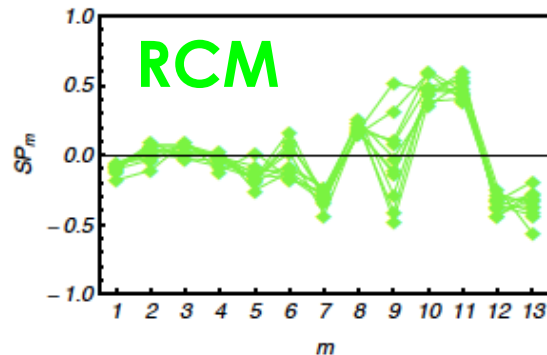
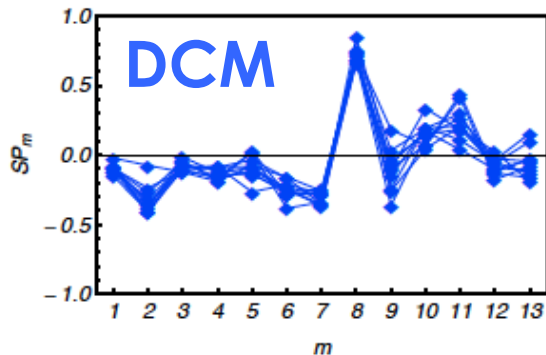
Significance profile:

$$SP_m \equiv \frac{z_m}{\sqrt{\sum_{m=1}^{13} z_m^2}}$$

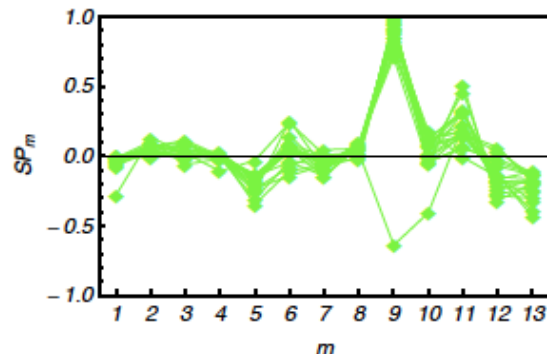
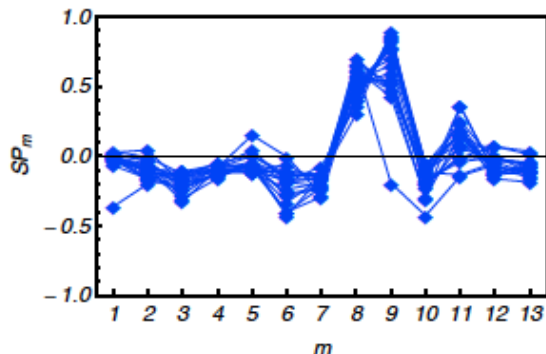


# 4 quasi-stationary sub-periods

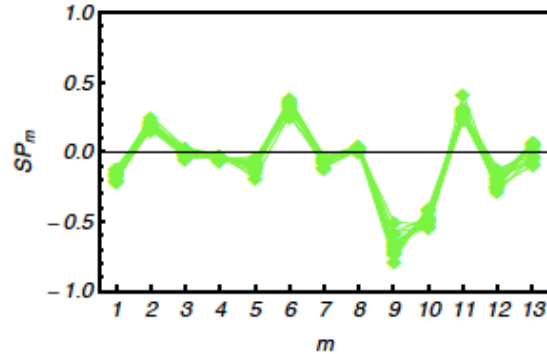
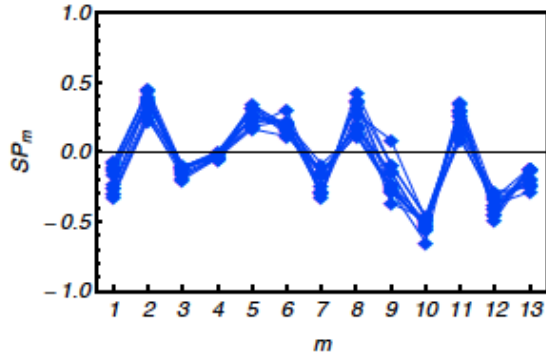
1998Q1-2000Q2.  
(stationarity)



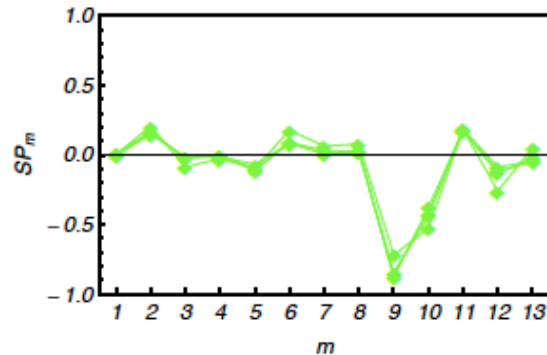
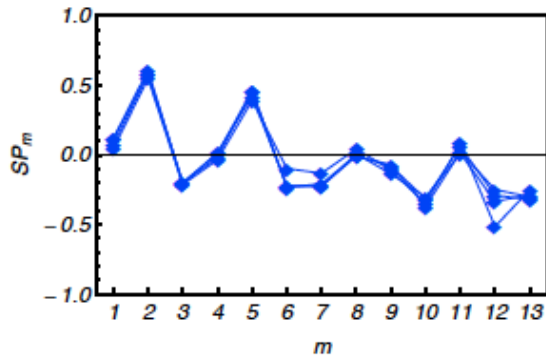
2000Q3-2004Q4  
(cyclic anomaly)



2005Q1-2007Q4  
(pre-crisis)

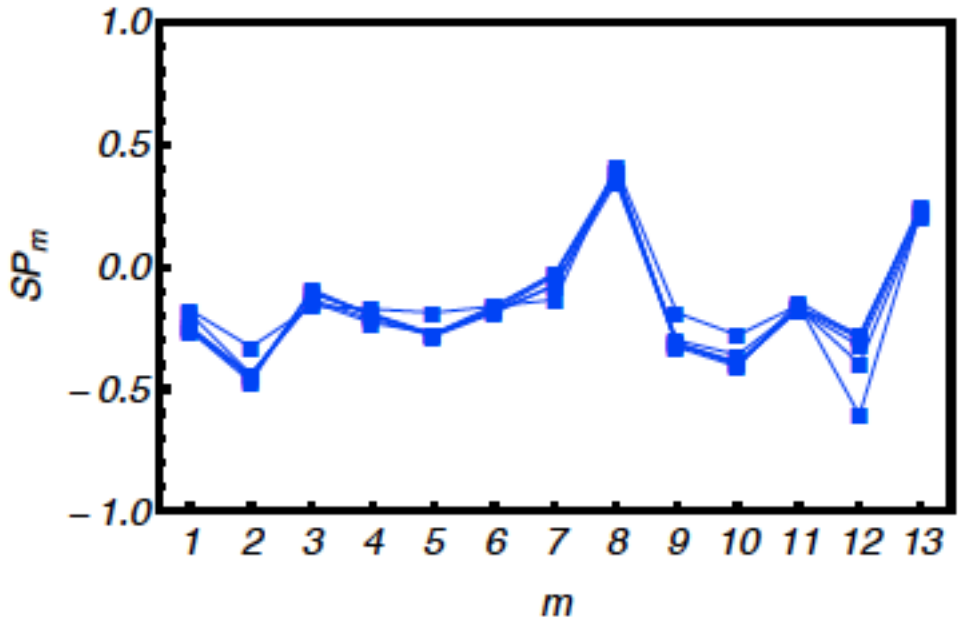


2008Q1-2008Q4.  
(crisis)

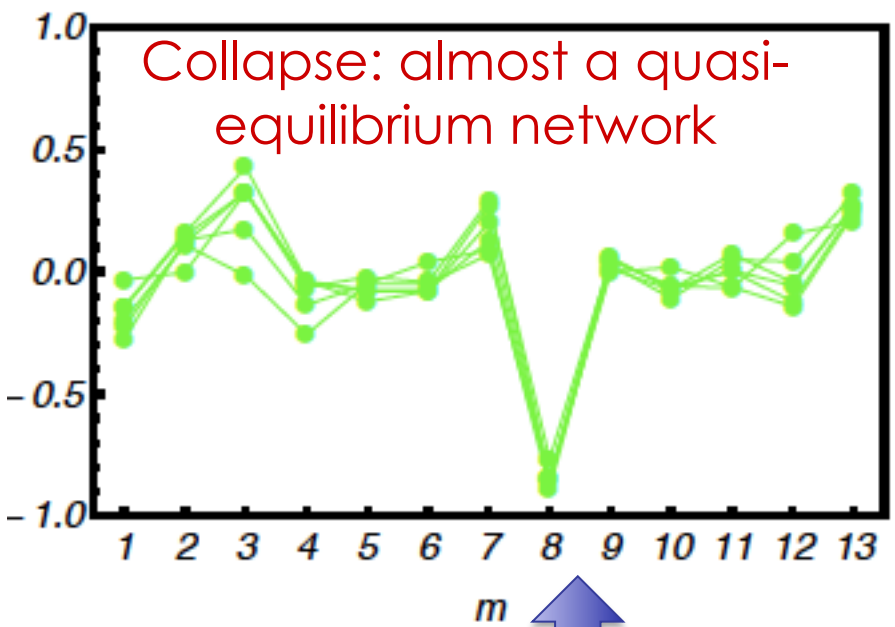


# Comparison with International Trade Network: quasi-stationary all the way (1950-2000)

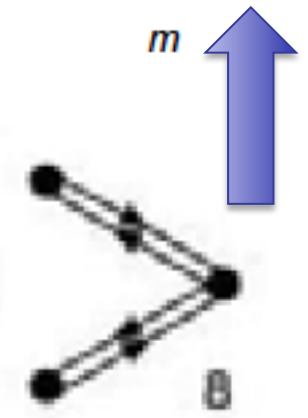
DCM



RCM



Only unexplained pattern  
(as expected from  
Granovetter's argument):



# In/out equilibrium: take-home messages

- **Maximum-Entropy** Ensembles are powerful to **check for (quasi-)equilibrium** and identify the **“right” constraints**;
- Economic networks are not well replicated/modeled without imposing **local topological constraints** (=“right” constraints);
- The **international trade network** is largely at (quasi-)equilibrium, and can be modeled by **coupling the GDP and/or distances to the right constraints** (improving upon gravity model);
- **Interbank networks at (quasi-)equilibrium** can be **reconstructed** reliably (along with their **systemic risk**) from the right constraints
- **As crises approach**, interbank network reconstruction is unreliable and actually **prevents** from detecting early-warning signals;
- Still, the comparison with the “right” (quasi-)equilibrium model is **crucial to create the early-warning signal** itself.