Endogenous fluctuations with variable money velocity of circulation

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Outline

Motivation

- This paper considers an infinite horizon economy with productive labor and with a partial Cash-In-Advance (CIA) constraint whose amplitude depends upon the amount of consumption bought.
- The inverse of the amplitude is the money velocity of circulation. In our framework: variable and endogenous.
- Under a complete and a constant CIA constraint local indeterminacy, endogenous cycles and sunspot equilibria require a high degree of intertemporal complementary in consumption.
- Too large to fit standard estimates (Abel, 1985; Bloise et al., 2000).

Motivation...

- The existence of sunspot cycles based on self-fulfilling expectations is an alternative explication of macroeconomic instability to the real business cycle theory.
- Sunspot cycles are driven by changes in expectations about the fundamentals.
- The analysis of cycles derived from agents' beliefs is built on the idea of sunspot equilibria.

Motivation...

- Bosi and Magris (2003), Bosi et al. (2005a; 2005b; 2007) establish that the amplitude of the CIA constraint on consumption expenditures has a great impact on the occurrence of endogenous fluctuations.
- In particular they show that as soon as the constraint is relaxed, endogenous fluctuations occur for degree of substitutability in intertemporal consumption larger and larger.
- There exists a lower bound for the liquidity constraint such that for amplitudes of the same constraint lower than it, local indeterminacy and sunspot fluctuations are bound to prevail for whatever parameters configuration.

Motivation...

- A partial CIA constraint is not a merely ad hoc device but is fully justified on an empirical ground, since in the real world the velocity of money is not constant and changes overtime.
- In the real world, only a share of expenditures must be paid cash. This is in particular true for transaction involving large amounts of money (e.g. real estate sector).

Motivation...

- These results are counter intuitive: the scope for sunspot fluctuations should decrease with the relaxation of the market imperfection and not increase.
- This should result of the role played by the credit cycle.
- Sunspot fluctuations become a pervasive phenomenom along with the reduction of the CIA constraint.

Our approach

- In our model we show that accounting for an endogenous CIA constraint has a lot of implications in terms of the occurrence of local indeterminacy and endogenous fluctuations (even chaotic dynamics).
- We establish, in terms of the elasticity of the amplitude of the CIA constraint with respect to consumption, the critical thresholds delimiting the different dynamics arising.
- The dynamics obtained are richer than in the case in which the amplitude of the CIA constraint is assumed to be constant, configuration that in our framework boils down to a particular case.

Intuition: Amplitude of the liquidity constraint is high

- Bosi et al. (2005) offer an intuition of the mechanism under the hypothesis that the latter is constant and exogenous.
- Suppose that the economy is in period t at the steady state and try to construct an alternative equilibrium in which agents anticipate a fall in next period price level.
- This implies that consumption in period t+1 increases.

Intuition: Amplitude of the liquidity constraint is high

- If the amplitude of the CIA constraint is close to one, under the Gross Substitutability Assumption, almost the whole labor income in period t is invested in money balances to buy next period (cheaper) consumption good, and the main arbitrage involves period t leisure and period t+1 consumption.
- It follows, that leisure in period t is driven down (labor supply is driven up) although less than in a one-to-one relationship with the increase of period t+1 consumption.
- Since at equilibrium labor equalize consumption, then also current consumption must increase. Such a condition requires a small fall in period t price, lower than the fall of the next period.
- This will induce an explosive dynamics which will violate the transversality condition, and thus it does not represent a possible equilibrium.

Intuition: Amplitude of the liquidity constraint is low

- Here, in correspondence to the expected increase in period t+1 consumption, agents will react by substituting period t labor with period t+1 labor, since now a large share of period t+1 consumption is financed out of period t+1 labor income.
- In particular, the lower the amplitude of the CIA constraint, the larger the contraction of period t+1 labor supply.
- Therefore, if the amplitude of the liquidity constraint is low enough, in order to preserve equilibrium period t price level must be sharply driven up in order to reduce current consumption.
- As a consequence, the system will move back towards its steady state, although following an oscillatory path.

Intuition in our approach

Since we assume that the amplitude of the liquidity constraint is a function (increasing or decreasing) of the amount of consumption bought, the effects described are either magnified or reduced.

Intuition: the amplitude of the liquidity constraint reacts negatively with the amount of consumption bought

- Consider that the amplitude of the liquidity constraint is low and that it reacts negatively with the amount of consumption bought.
- Here, in correspondence to the expected increase in period t+1 consumption, agents will react by substituting period t labor with period t+1 labor in away even more ”aggressive” than in the case the constraint were constant, since now a large share of period t+1 consumption is financed out of period t+1 labor income.
- In particular, the lower the amplitude of the CIA constraint, the larger the contraction of period t+1 labor supply. Therefore, if the amplitude of the liquidity constraint is low enough, in order to preserve equilibrium period t price level must be sharply driven up in order to reduce current consumption.
- As a consequence, the system will move back towards its steady state, although following an oscillatory path.

Intuition: the amplitude of the liquidity constraint reacts positively with the amount of consumption bought

- Consider that the amplitude of the liquidity constraint is low and that it reacts positively with the amount of consumption bought.
- Here, in correspondence to the expected increase in period t+1 consumption, agents will react by substituting in a lower way period t labor with period t+1 labor than in the case the constraint were constant. A large share of period t+1 consumption is financed out of period t labor income.
- Therefore, if the amplitude of the liquidity constraint is low enough, in order to preserve equilibrium period t price level is driven up only lightly in order to reduce current consumption. As a consequence, it will be hard for the system to move back towards its steady state.

2. The model

2.1 Central Bank and Monetary Policy

Central Bank and Monetary Policy

- The Central Bank issues money against the purchase of government bonds through open market operations.
- The budget constraint of the Central Bank is

$$B_{t+1}^{CB} = I_t B_t^{CB} + M_{t+1} - M_t \tag{1}$$

- with B_{t+1}^{CB} the amount of nominal government bonds, and M_{t+1} the stock of nominal balances
- The central bank creates money, and it gives this money as a lump sum transfer.

$$M_t = (1 + \hat{\sigma}) M_{t-1} = \sigma M_{t-1} \tag{2}$$

- with M_t the total supply of money, $\hat{\sigma}$ the rate of money creation.
- T_t is the lump sum transfer received by the agents living at period t. This transfer is financed by money creation:

$$\hat{\sigma} M_t = T_t \tag{3}$$

2.2 The Government and the Fiscal Policy

The Government and the Fiscal Policy

- The government budget constraint relative to period t is therefore given by
$$B^g_{t+1} = p_t g_t + I_t B^g_t. \tag{4}$$
- g_t denote the government public spending in real terms in period t, $I_t \equiv 1 + i_t$ be the nominal interest factor in period t, i_t being the nominal interest rate relative to the same period, B^g_{t+1} the nominal amount of safe government bonds issued in period t and p_t the price of the (unique) consumption good.
- Let us assume in addition that the initial amount of nominal government bonds issued in period zero is $B^g_0 > 0$.

2.3 Household

Household

- We consider an infinite horizon discrete time economy populated by a constant mass of agents whose size is normalized to one.
- The preferences of the representative agent:

$$\sum_{t=0}^{+\infty} \beta^t \left[u\left(c_t\right) - Av\left(l_t\right) \right] \tag{5}$$

- where c_t is the unique consumption good, M_t the money balances, l_t the labor supply, p_t the price of consumption good, $\beta \in (0, 1)$ the discount factor and $A > 0$ a scaling parameter.

Household...

- When maximizing the utility agents must respect the dynamic budget constraint:

$$p_t c_t + M_{t+1} + B_{t+1} = M_t + (1 + i_t) B_t + w_t l_t + T_t \tag{6}$$

- We suppose that agents must pay cash at least a share $\chi(c_t) \in (0, 1]$ for their consumption purchases:

$$\chi(c_t) p_t c_t \leq M_t \tag{7}$$

$$\chi'(c_t) > 0,$$
$$\chi'(c_t) < 0$$

Household...

- From the first-order conditions, we obtain the following equations:

$$\beta u'(c_{t+1}) = A(1-\chi_t)\beta v'(l_{t+1}) + A\chi_t \sigma v'(l_t)\frac{p_{t+1}}{p_t} \tag{8}$$

$$u'(c_t) = \beta u'(c_{t+1})\frac{\chi_t(1+i_t)+1-\chi_t}{\chi_t \frac{p_{t+1}}{p_t} + (1-\chi_t)(1+i_{t+1})^{-1}\frac{p_{t+1}}{p_t}} \tag{9}$$

2.4 Equilibrium and Steady state

Equilibrium

- We assume a linear technology : $y_t = l_t$.
- Equilibrium in the good market is $y_t = l_t = c_t$
- Equilibrium in the money market requires $p_{t+1}/p_t = m_t\sigma/m_{t+1}$.
- The technology is linear in labor, one has that the real wage is 1.

Equilibrium...

- Let us define $U = yu'$ and $V = v'y$
- An intertemporal interior equilibrium with perfect-foresight is a sequence $\{y_t\}_{t=0}^{\infty}$, $\forall t \geq 0$, satisfying:

$$\beta U(y_{t+1}) = A\left[1 - \chi(y_t)\right]\beta V(y_{t+1}) + A\frac{\chi(y_t)^2}{\chi(y_{t+1})}\sigma V(y_t) \tag{10}$$

Steady state

- A steady state is given by:

$$\beta U(y) = A\left[1 - \chi(y)\right]\beta V(y) + A\chi(y)\sigma V(y) \tag{11}$$

- We now calibrate a particular solution of (11) with $y = 1$

$$A = \frac{U(y)}{V(y)[1-\chi(y)+\chi(y)\beta^{-1}\sigma]} \tag{12}$$

- To ensure that the CIA constraint binds in a neighborhood of the steady state we assume that the money growth factor is strictly larger than the discount factor: $\sigma > \beta$.
- The stationary nominal interest rate is $i = \beta^{-1} - 1 > 0$: bonds dominate money in terms of returns.

Concluding remarks

- The paper studies an infinite horizon economy with productive labor with a partial Cash-In-Advance (CIA) constraint whose amplitude depends upon the amount of consumption bought.
- In our model we show that accounting for an endogenous CIA constraint has a lot of implications in terms of the occurrence of local indeterminacy, endogenous fluctuations and even chaotic dynamics.
- We establish, in terms of the elasticity of the amplitude of the CIA constraint with respect to consumption, the critical thresholds delimiting the different dynamics arising.

Concluding remarks

- An interesting question concerning our model is under which specification for the amplitude of the liquidity constraint aggregate welfare is maximized. The answer is quite intuitive. Since the liquidity constraint, by bounding agents to hold money balances, entails a cost expressed by the nominal interest rate, one should expect that a reduction of the amplitude of the CIA constraint allows avoiding a share of such a cost. It follows that a liquidity constraint decreasing in the amount of consumption bought should increase agents' welfare.
- A straightforward generalization of the model should account for capital accumulation, an open economy.

Concluding remarks

Thank you for your attention!