

# A topology of world equity markets, 1960-2015

Cécile Bastidon<sup>1</sup>   Antoine Parent<sup>2</sup>

<sup>1</sup> Université de Toulon, LEAD (EA 3163)

<sup>2</sup> IEP Lyon, LAET CNRS (UMR 5593)



# Contents

## Introduction

1. Sample and methodology
  - 1.1. Characteristics of the sample
  - 1.2. Networks, graphs and subgraphs
  
2. Empirical results
  - 2.1. Topological representations
  - 2.2. Topological characteristics
  - 2.3. Summary

## Concluding remarks

# Contents

## Introduction

1. Sample and methodology
  - 1.1. Characteristics of the sample
  - 1.2. Networks, graphs and subgraphs
  
2. Empirical results
  - 2.1. Topological representations
  - 2.2. Topological characteristics
  - 2.3. Summary

Concluding remarks

# Topological representations and financial integration

World equity markets in the long run, using the literature on topological representations of assets prices (*MANTEGNA, 1998; BONANNO et al., 2000; TUMMINELLO et al., 2008*):

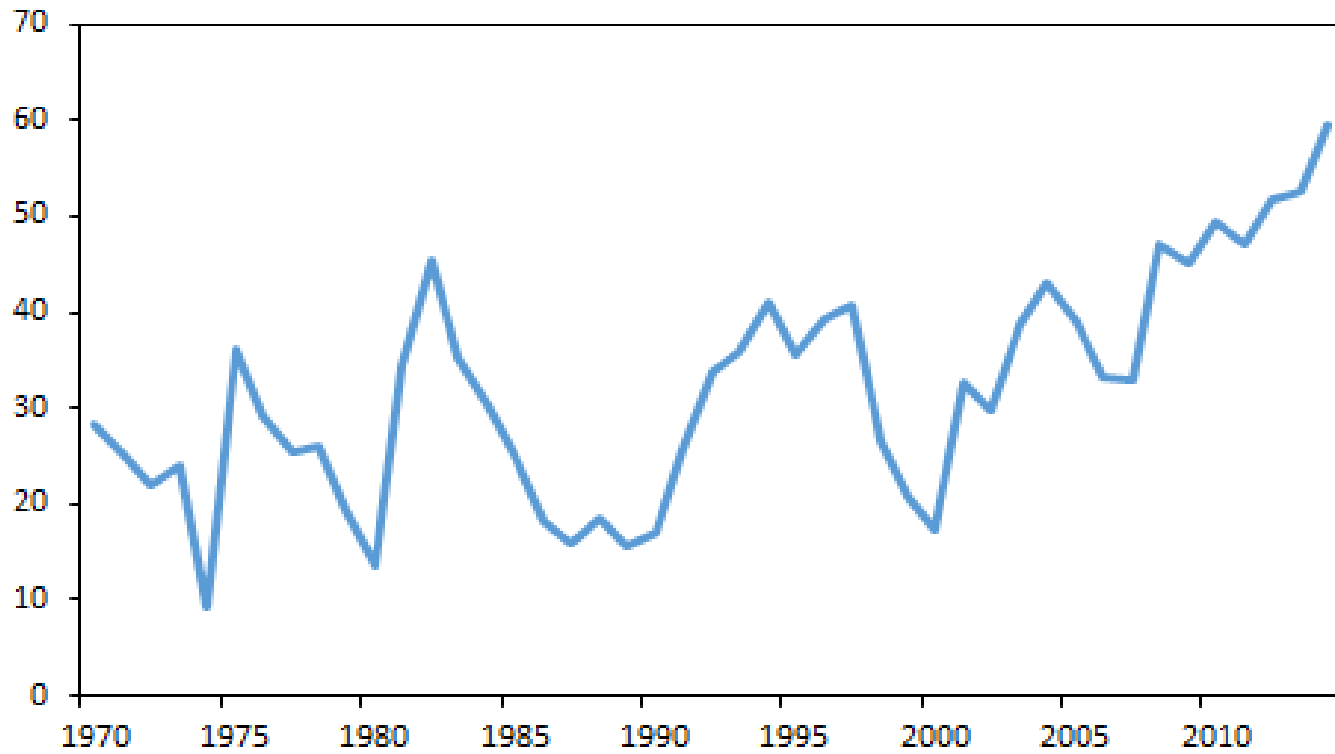
- 1<sup>st</sup> contribution: integration of world capital markets as complex systems
- 2<sup>nd</sup> contribution: historical approach over 5 decades

Usual measures of capital markets integration (*OBSTFELD and TAYLOR, 2004*):

- ratios of stocks of foreign investment or foreign assets to GDP
- ratio of the level of domestic investment to domestic savings (*FELDSTEIN and HORIOKA, 1980*)
- co-evolution of domestic and foreign interests rates
- *etc.*

Increasing integration of capital markets since 1945, limited to the *intensive margins*:

- Lucas Paradox (*LUCAS, 1990*): developing economies of the post-colonial era characterised by very low flows of international capital
- peculiar dynamics of foreign direct investment during the 2000s



**Figure 1: FDI flows to emerging and developing countries**  
% of the world total, source : UN

# Contents

## Introduction

1. **Sample and methodology**
  - 1.1. **Characteristics of the sample**
  - 1.2. Networks, graphs and subgraphs
2. Empirical results
  - 2.1. Topological representations
  - 2.2. Topological characteristics
  - 2.3. Summary

## Concluding remarks

# Price series

Database: FRED

Indices: 32 countries (last subperiod)

5 sub-periods: 1960s, 1970s, 1980s, 1990s, 2000-2015

- Different break dates (*BEKAERT et al.*, 2002) for different countries
- Price series: peculiarity of the 1980s and 1990s

	Mean	Median	Minimum	Maximum	Standart dev.
1960s	1,30E+02	1,16E+02	6,81E+01	4,17E+02	4,44E+01
1970s	1,20E+02	1,03E+02	3,87E+01	1,10E+03	7,59E+01
1980s	2,94E+03	1,94E+02	3,58E+01	1,71E+05	1,54E+04
1990s	8,30E+07	1,23E+02	3,48E+01	7,82E+09	6,10E+08
2000-2015	1,78E+02	1,37E+02	1,02E+01	9,69E+02	1,42E+02

(observations 1950/01 - 2014/12, base 100 at the beginning of each sub-period)

**Table 1: Descriptive statistics by sub-periods**

# Contents

## Introduction

1. **Sample and methodology**
  - 1.1. Characteristics of the sample
  - 1.2. **Networks, graphs and subgraphs**
  
2. Empirical results
  - 2.1. Topological representations
  - 2.2. Topological characteristics
  - 2.3. Summary

## Concluding remarks



# Large economic / financial networks

Topological approach of complex systems:

- Network representations starting from correlation matrices
- Networks: nodes, edges (distances)

Field of finance:

- Complete networks (e.g. equity markets...):  $n^2$  edges
- Large incomplete networks (e.g. interbank markets...):  $<n^2$  edges but large
- Need to isolate the economically meaningful edges

## Minimal spanning tree (MST):

- (Unique) subgraph of a given network
- Subdominant ultrametric

## Hierarchical tree (HT):

- Classification
- HT obtained by the *single linkage* or *nearest neighbor* method: same branches as the MST
- Complementarity of the MST and HT

# From correlations to the MST and HT

Correlations matrices of log differences of monthly index price data.

Correlation coefficient of variables  $i$  and  $j$  :

$$\rho_{ij} = \frac{\text{Cov}(IDV_i, IDV_j)}{\sigma_{IDV_i}, \sigma_{IDV_j}}$$

Distance matrices:  $d(i, j) = 1 - \rho_{ij}^2$  satisfies the axioms of an Euclidian metric:

1.  $d(i, j) = 0$  if and only if  $i = j$
2.  $d(i, j) = d(j, i)$
3.  $d(i, j) \leq d(i, k) + d(k, j)$

Subdominant ultrametric of the distance matrix: MST (Kruskal's algorithm), HT (nearest neighbor method):

$$d(i, j) \leq \sup(d(i, k), d(k, j))$$

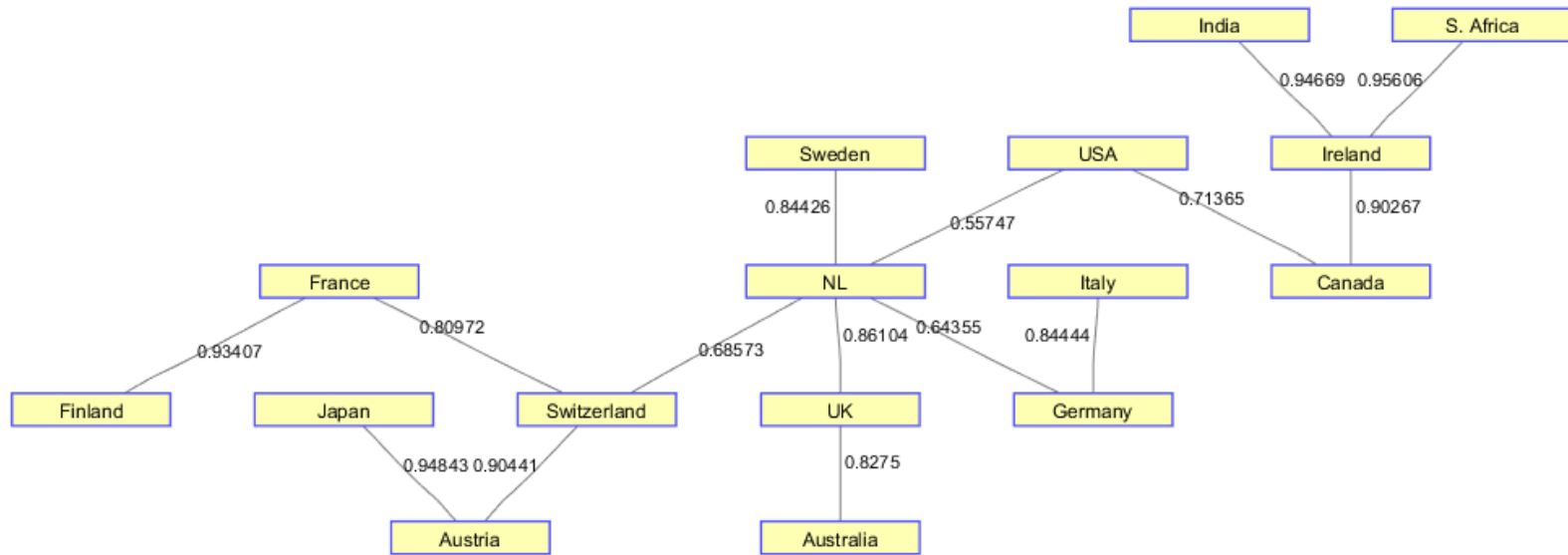
# Contents

## Introduction

1. Sample and methodology
  - 1.1. Characteristics of the sample
  - 1.2. Networks, graphs and subgraphs
  
2. Empirical results
  - 2.1. Topological representations
  - 2.2. Topological characteristics
  - 2.3. Summary

## Concluding remarks

# Topology, 1960s



**Figure 2a: Minimal spanning tree, 1960s**

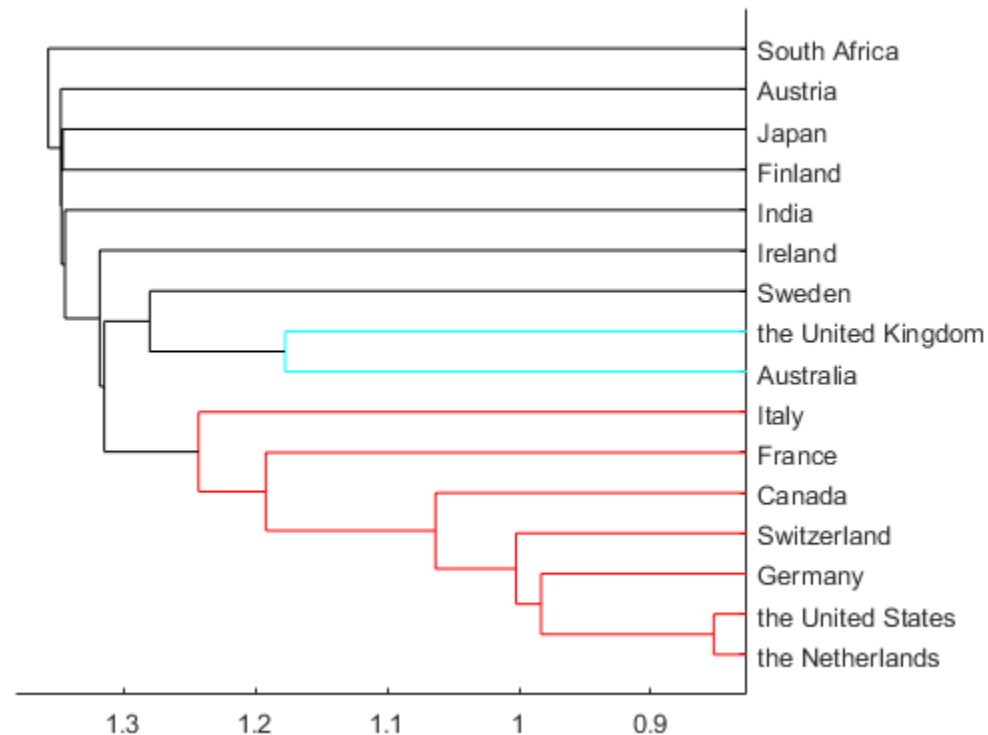
## MST:

- Historical logic (UK and former British colonies)
- Geographical logic (Continental Europe)
  - Central place of NL in the Continental Europe subtree (all sub-periods)

## HT:

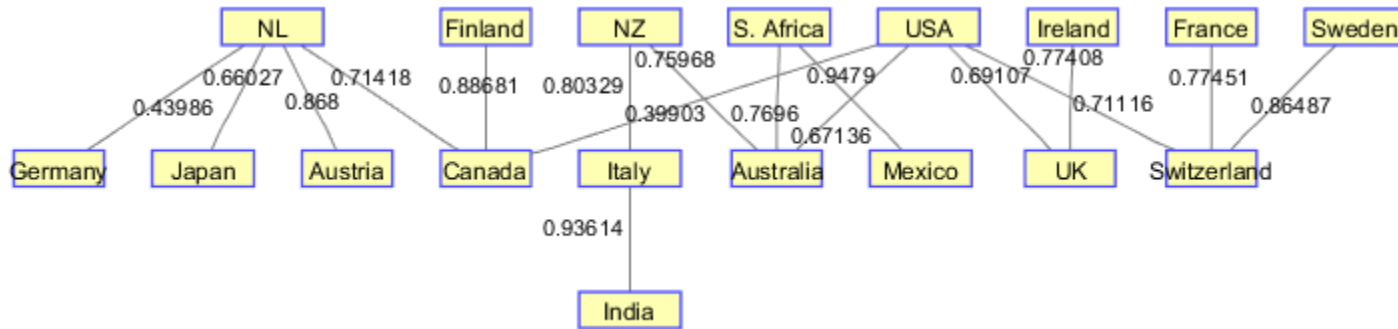
- Narrow group of highly integrated markets (in order of increasing distances, Netherlands, USA, Germany, Switzerland and Canada)

# Topology, 1960s



**Figure 2b: Hierarchical tree, 1960s**

# Topology, 1970s



**Figure 3a: Minimal spanning tree, 1970s**

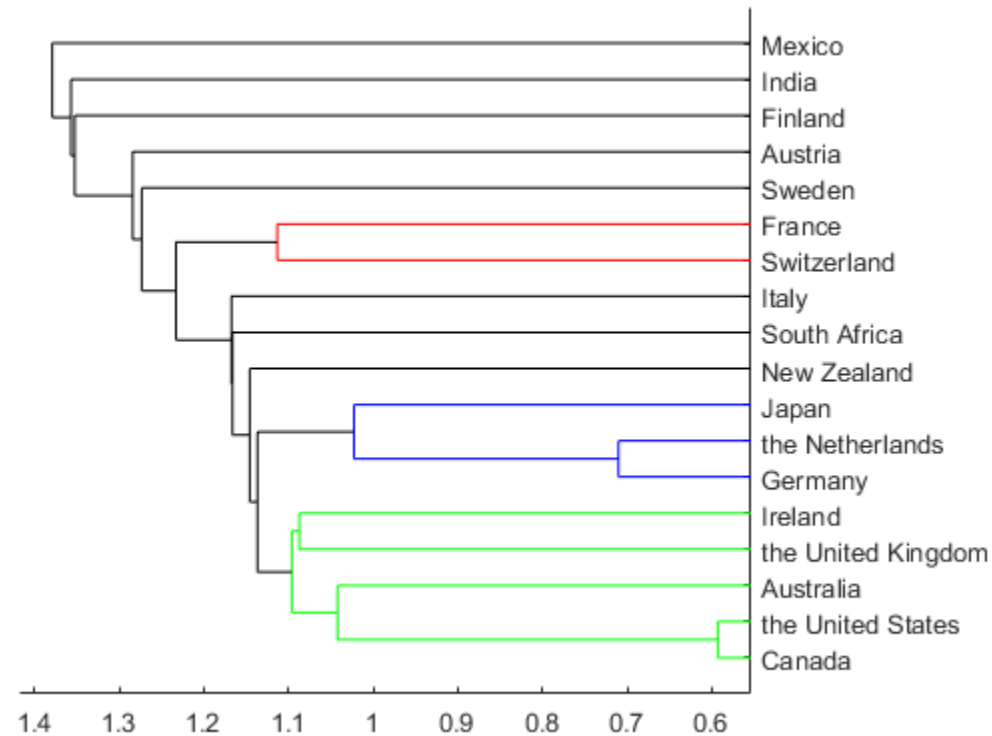
MST:

- 2 nodes (NL, USA) associated with an atypically large number of edges
- No obvious historical or geographical logic in the composition of the associated sub-trees
- Atypical proximity of two pairs of indices: Netherlands - Germany and especially USA - Canada

HT:

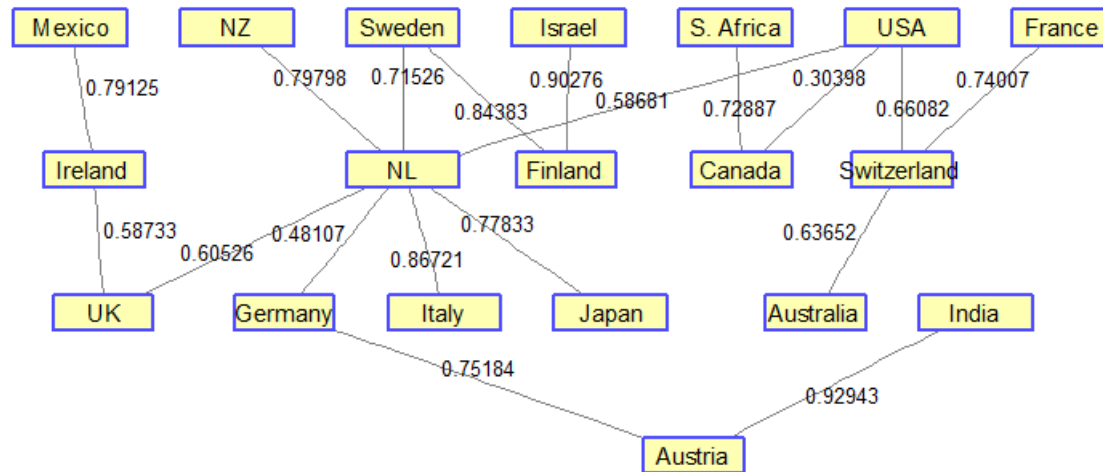
- Confirms the atypical proximity of Netherlands - Germany and USA - Canada

# Topology, 1970s



**Figure 3b: Hierarchical tree, 1970s**

# Topology, 1980s



**Figure 4a: Minimal spanning tree, 1980s**

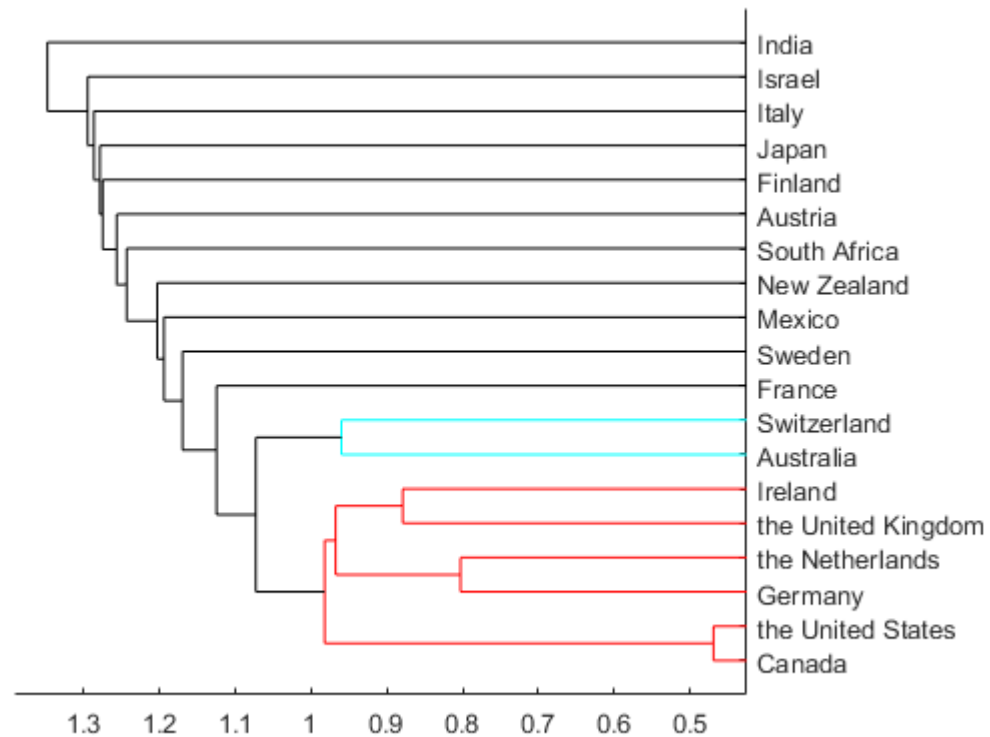
MST:

Roughly similar to the 1970s:

- 2 nodes (NL, USA) associated with an atypically large number of edges (4 and 4 in the 1970s, 6 and 3 in the 1980s)
- Atypical proximity of Netherlands - Germany and especially USA - Canada, confirmed by the HT

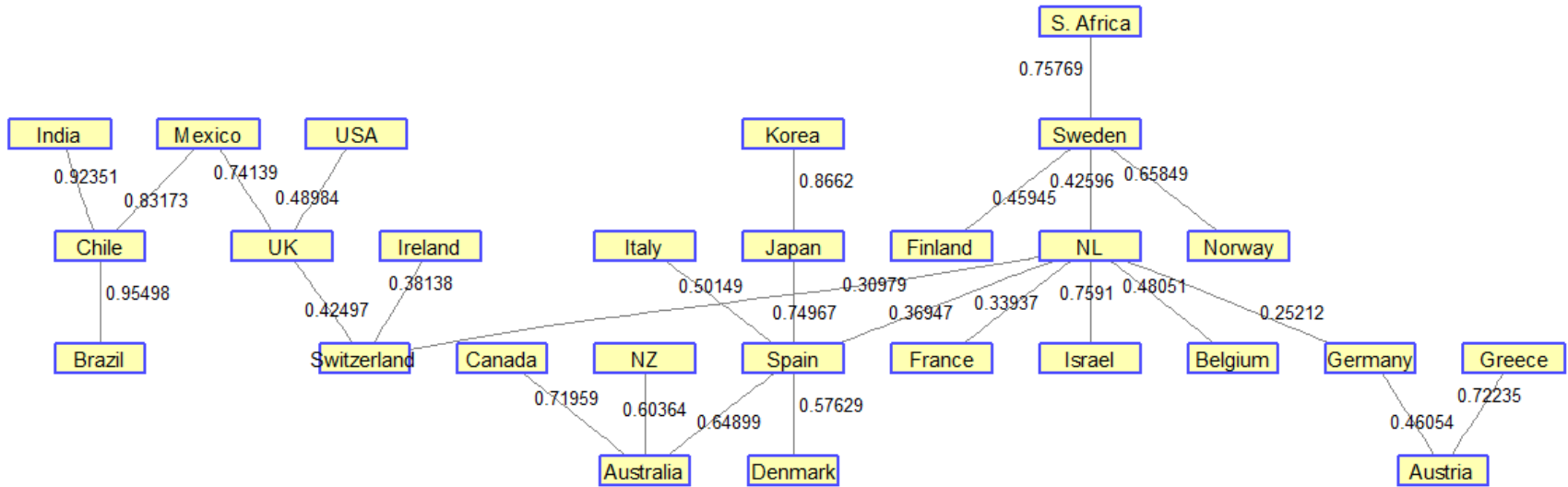


# Topology, 1980s



**Figure 4b: Hierarchical tree, 1980s**

# Topology, 1990s



**Figure 5a: Minimal spanning tree, 1990s**

MST:

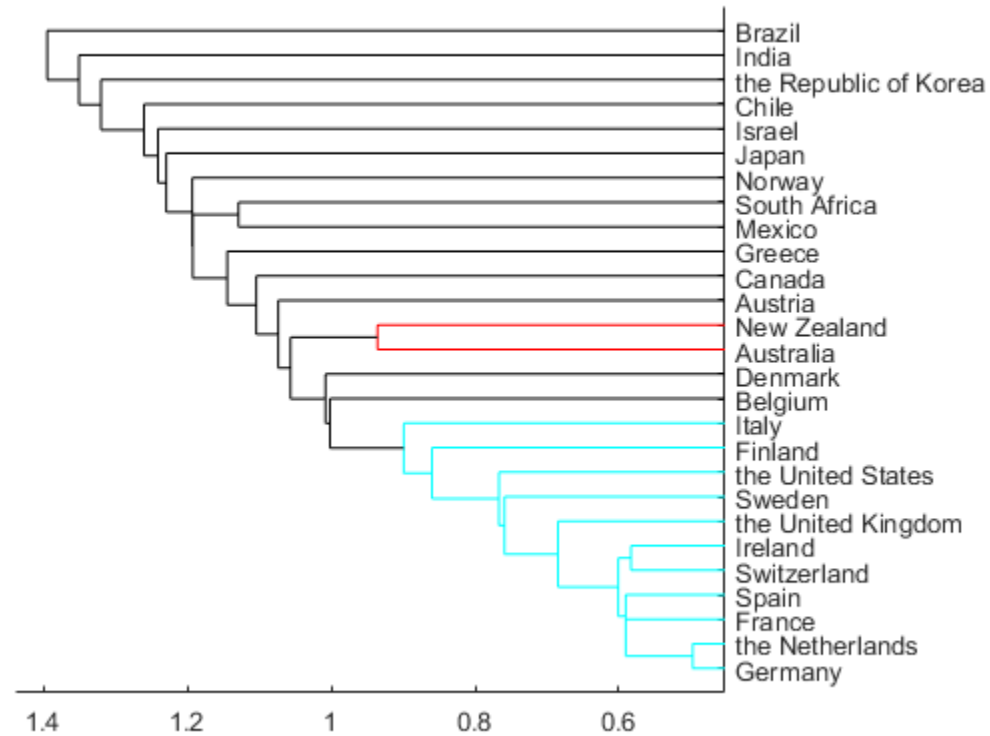
- Peculiar topological status of NL (7 edges, lowest distances of the sample)

HT:

3 groups:

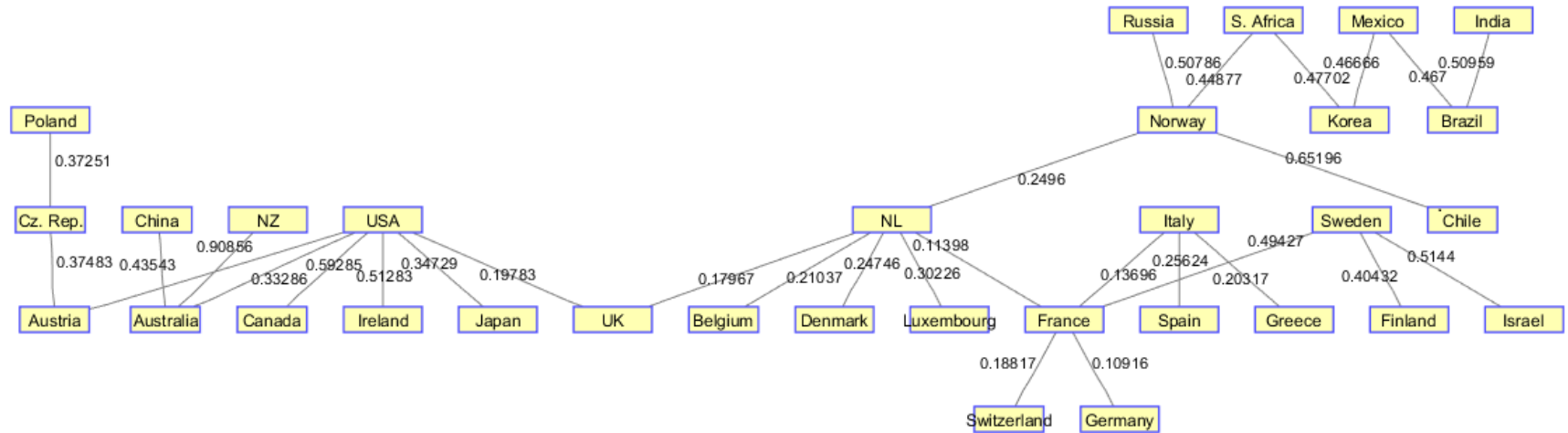
- Highly integrated advanced economies
- Intermediately integrated advanced economies
- Less integrated emerging economies (unchanged)

# Topology, 1990s



**Figure 5b: Hierarchical tree, 1990s**

# Topology, 2000-2015



**Figure 6: Minimal spanning tree, 2000-2015**

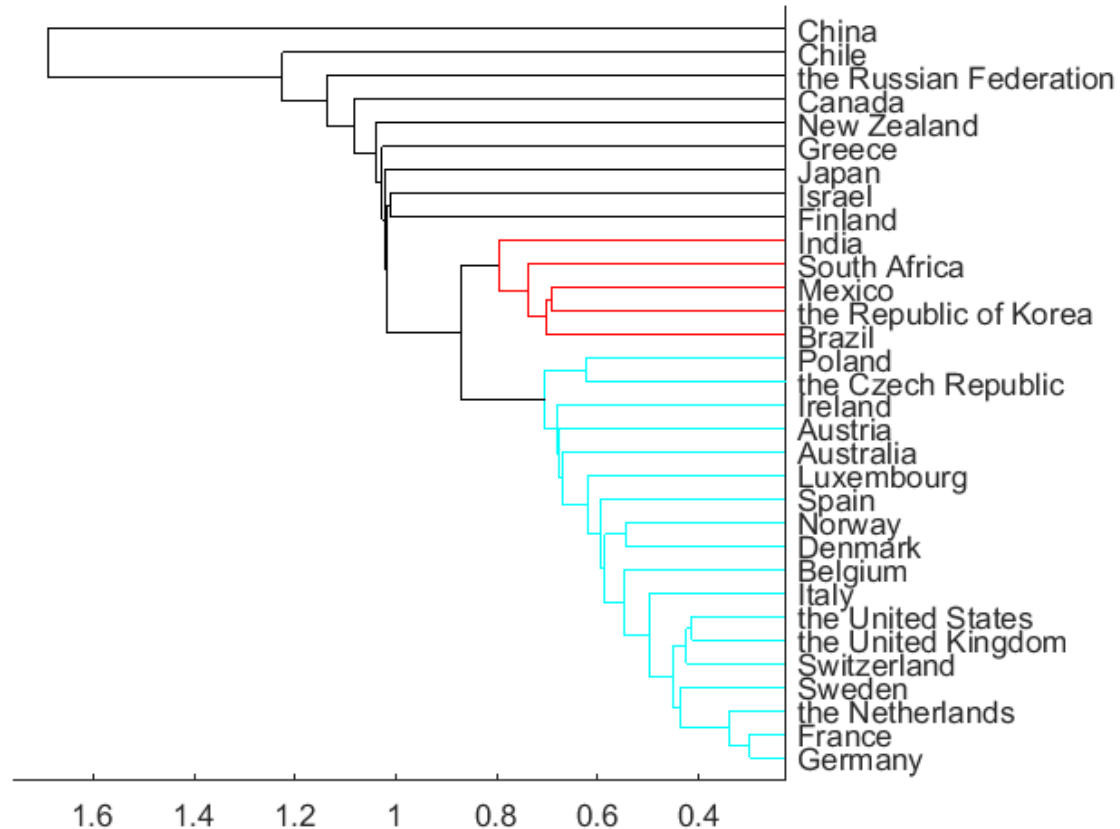
## MST:

- Organised around 2 nodes (NL and USA), linked by the UK
- Shortest distances of the sample in the corresponding sub-trees

## HT:

- New cluster of emerging economies with intermediate degrees of integration
- Less integrated economies: both emerging (Israel, Russia, Chile, China) and advanced economies (Japan, Greece, New Zealand, Canada)
- Integration at the extensive margins in progress ?

# Topology, 2000-2015



**Figure 6b: Hierarchical tree, 2000-2015**

# Contents

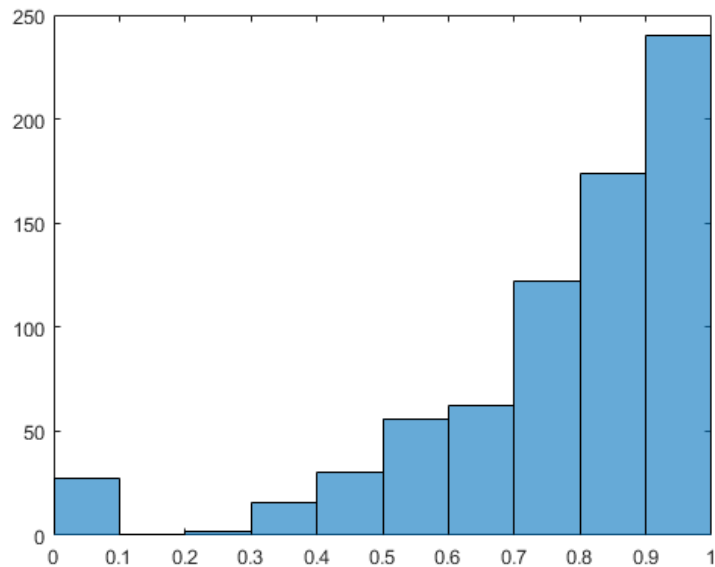
## Introduction

1. Sample and methodology
  - 1.1. Characteristics of the sample
  - 1.2. Networks, graphs and subgraphs
  
2. **Empirical results**
  - 2.1. Topological representations
  - 2.2. **Topological characteristics**
  - 2.3. Summary

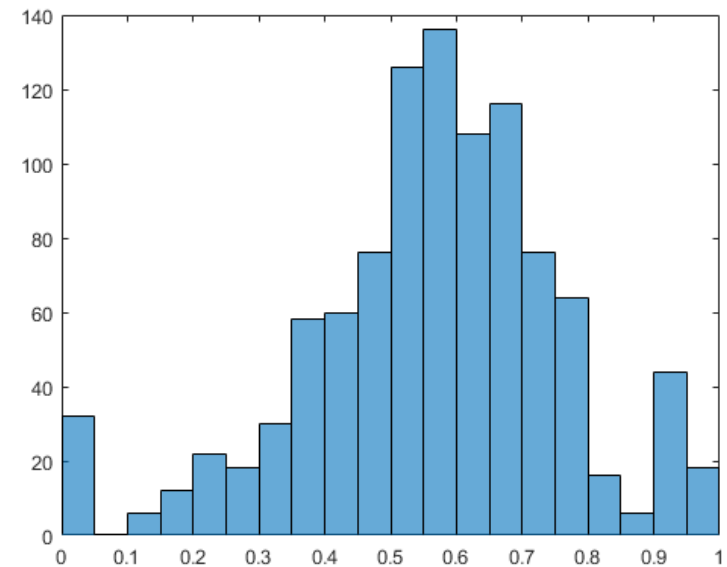
## Concluding remarks

# Histograms of distances matrices

**Fig 6a: Distances matrix, 1990s**

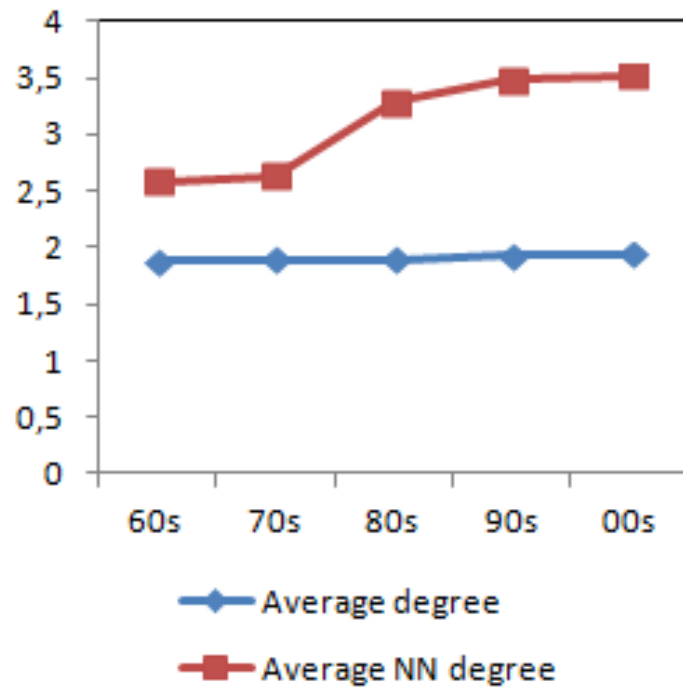


**Fig 6b: Distances matrix, 2000-2015**

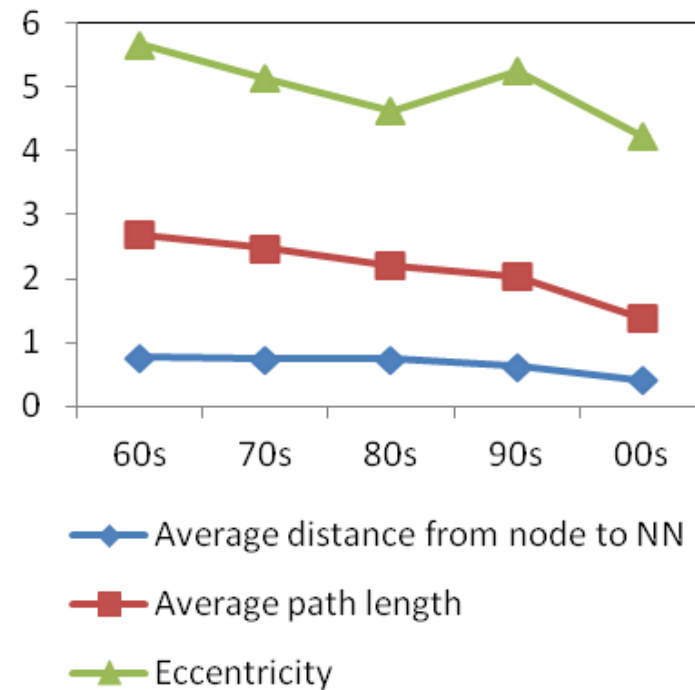


# Network statistics

**Fig 7a: Average degrees, average nearest neighbour degree**



**Fig 7b: Average strength, average path length, excentricity**





# Contents

## Introduction

1. Sample and methodology
  - 1.1. Characteristics of the sample
  - 1.2. Networks, graphs and subgraphs
  
2. **Empirical results**
  - 2.1. Topological representations
  - 2.2. Topological characteristics
  - 2.3. **Summary**

## Concluding remarks

# Main results

1. Lateness of the appearance in these topological representations of truly global markets, *i.e.* tripolar and characterised by a predominance of intermediate distances.
2. The USA acquire only at a late stage (in the 2000s) the dominant position which is expected (structure of trees, distances)
3. The only geographical or historical persistent logic over the sub-periods is the topological proximity of the equity markets of continental Europe.
4. Emerging economies in the 2000s split into two sub-groups.
  - 1<sup>st</sup> group: intermediate degree of integration
  - 2<sup>nd</sup> group: weakly integrated.

# Contents

## Introduction

1. Sample and methodology
  - 1.1. Characteristics of the sample
  - 1.2. Networks, graphs and subgraphs
2. Empirical results
  - 2.1. Topological representations
  - 2.2. Topological characteristics
  - 2.3. Summary

## Concluding remarks

# Research perspectives

1. The overall results justify the choice of the methodology: economically meaningful topological representations derived from long term series of equity markets indices.
2. A research perspective could be to build topological representations iteratively, in order to determine the periods as defined by the dates that generate significantly positive breakdown tests on the overall topological characteristics.