Endogenous fluctuations with variable money velocity of circulation

Francesco Magris*[1], Antoine Le Riche[2], and Antoine Parent[3]

[1]LEO – Université de Tours, university of Tours – Université de Tours, France
[2]LAET – Institut d'Études Politiques [IEP] - Lyon – Sciences Po Lyon, France
[3]LAET – Institut d'Études Politiques [IEP] - Lyon – France

Abstract

It is well known that growth model may be locally indeterminate, i.e. may possess a steady state such that the equilibrium dynamics converge toward it for infinitely many choices for the initial conditions, when some market imperfection is accounted for. This is the case when one assumes some external effect in production not mediated by markets, when participation to the latter is restricted, as it is the case in OLG models, or as a consequence of the frictions due to the introduction of money as a medium of exchange. When an equilibrium is indeterminate, it is possible to construct nearby stochastic sunspot equilibria, i.e. equilibria representing optimal responses to agents' revisions in their beliefs (Woodford, 1986; Grandmont et al. 1998). In this paper, within a Money-in–Utility Function model, we show that local indeterminacy, endogenous fluctuations and chaotic dynamics may emerge for degree of financial market imperfections arbitrarily close to zero. Such a result is rather innovating, since commonly it is supposed that such phenomena are more likely to emerge for high degrre of market imperfection, i.e. for robust departures from the First Welfare Theorem.
The extensions of the standard neoclassical growth models effectuated in order to account for the role played by money include the seminal Sidrauski (1967) and Brock (1974; 1975) models in which real balances enter into the instantaneous utility function besides to consumption good and, possibly, leisure. The inclusion of real balances in the utility function is motivated by the perception of money as one of many assets (some financial, some real) and the indirect utility money provides stemming from the liquidity allowing to carry out transactions and the possibility to save time compared to being illiquid. The money in the utility function approach can be viewed indeed as a device to account for the liquidity services money provides by reducing the transaction costs (e.g. shopping time models): as it is shown in Feenstra (1986) each transaction cost formulation is equivalent to an appropriate money-in-the-utility function specification. Also the cash-in-advance constraint approach (Clower, 1967; Stockman, 1981; Abel, 1985, Svensson, 1985; Lucas and Stokey, 1987; Cooley and Hansen, 1989) which compel agents to purchase the consumption good out of money balances previously accumulated falls within a specification of the utility function in which consumption and real balances are assumed to be perfect complements. Another approach, as in Benhabib and Farmer (2000) suppose that money reduces the selling costs and can then be viewed as a productive input, besides, e.g., physical capital. They show that a sufficient condition for indeterminacy is that money is sufficiently productive. In absence of transaction or selling costs, money can be positively valued only in OLG models (Samuelson 1958;

*Speaker

Gale, 1973; Azariadis, 1981; Grandmont, 1985) where merely *fiat* money is delivered from the old generation to the young one and can be therefore viewed as a speculative bubble (Tirole, 1985).

Brock (1974; 1975) is the first to discuss the possibility of indeterminacy in a model that has real balances in the utility function. He shows how self-fulfilling hyperinflationary and deflationary equilibria can occur and under which conditions to rule out such phenomena. Calvo (1979) within a money-in-the-utility function approach shows that indeterminacy is most easily obtained when changes in the stock of real balances have large effects on output, while Matsuyama (1991) proves that cyclical and even chaotic equilibria can be obtained when the rate of money growth supply is high. Farmer (1997) consider a RBC model with money in the utility function and calibrates the model to fit the first moments of U.S. by choosing a parametrization of utility for which the model admits the existence of indeterminate equilibria in order to explain the monetary propagation mechanism. The mechanism that generates indeterminacy is that a small increase in real balances must be associated with a big increase, in equilibrium, of labor allocated to production.

In our model there is an infinitely lived representative agent whose utility function includes consumption, real balances and leisure. Besides money, there are safe bonds whose initial endowment is zero and whose nominal interest rate is strictly positive, implying that money is dominated by them in terms of returns. Our model differ from analogous ones by the fact that in the utility function real balances do appear multiplied by a parameter included between zero and infinite. Such a parameter can be viewed as the inverse of the degree of financial imperfection: a large value of it reflects the fact that the liquidity services provided by money are very sensitive to the amount of the real balances held. A larger value of such a parameter reflects indeed underlying diminishing transaction costs and as a consequence, for a given level of real balances, a larger marginal utility of money. We make abstraction of the emergence of self-fulfilling hyperinflationary equilibria studied in Brock (1974) and concentrate the attention on the monetary stationary solutions and their stability. We show that when labor is supplied rather elastically, in contrast to Farmer (1997), indeterminacy occurs for a wide range of the degree of financial imperfection values arbitrarily close to zero. As a matter of fact, indeterminacy appears first through a transcritical bifurcation and then through a flip bifurcation: in the first case, as shown in Grandmont (2008) and Bosi and Ragot (2011), one will assist to a change in stability between two nearby steady states and, in the second case, a 2-period cycle (stable or unstable, according to the direction of the bifurcation) will arise arbitrarily close to the stationary solution. On the other hand, when labor is supplied rather inelastically, the transcritical bifurcation is ruled out, meanwhile indeterminacy is bound to prevail through a flip bifurcation for a whole range of the degree of financial imperfection unbounded away from zero. We also show that chaotic dynamics may emerge for a wide range for the parameters configuration, again provided that the degree of financial market imperfection is set low enough.
The fact that chaos and indeterminacy arises more easily when the degree of financial imperfection is lower seems to be counterintuitive since it is usual to see that indeterminacy is more likely to occur when the market imperfection are important, and to do not shrink when the latter are reduced. However, our results confirm that of Bosi, Cazzavillan and Magris (2005) in which a fractional cash-in-advance on consumption purchases is assumed; they show that indeterminacy (and even chaotic dynamics) occurs for a wide range of the structural parameters as soon as the share of the consumption good to be paid cash is set sufficiently low. The mechanism leading to indeterminacy is, roughly, the following. Assuming for sake of simplicity an inelastic labor supply, let us suppose that the system is at the beginning at its steady state and that agents anticipate, say, an increase in the price level of the following period. Accordingly, they will react by decreasing the desiderated amount of money balances held at the end of the foregoing period and by increasing the current amount of nominal balances. But, if money balances are not too much substitutable across time and if such an effect is magnified by the lower degree of financial imperfection, the investment in money balances will decrease only lightly. At the same time, the current price level will decrease sharply in order to re-establish equilibrium in money market (we assume that the money supply is constant), giving rise to an oscillatory dynamics with decreasing amplitudes

and therefore converging back to the stationary solution.